Fiber Optic Communications

Ch 3. Optical receivers

Contents

Optical receivers

- p–i–n diodes
- Avalanche diodes
- Receiver design
- Receiver noise
  - Shot noise
  - Thermal noise
- Signal-to-noise ratio

Optical receivers

- The purpose of a traditional receiver for OOK is:
  - Convert the optical signal into an electrical signal
  - Recover the data by:
    - Doing clock recovery
    - Performing decisions on the obtained signal
- In state-of-the-art coherent receivers, additional functionality is performed in digital signal processing (DSP)
  - Electronic dispersion compensation (EDC)
  - Adaptive equalization
  - Phase synchronization
- This lecture is about OOK systems
  - Necessary to know about...
  - ...and still common

Photodetectors

- The most critical component is the photodetector
  - Converts the optical signal to an electrical current
- We want these components to have:
  - High sensitivity
  - Fast response time
  - Low noise
  - High reliability
  - Size compatible with fibers
- This means that semiconductor materials are exclusively used
  - Photons are absorbed and generate electron–hole (e–h) pairs
  - This produces a photo-current.
- Basic requirement: The detector material bandgap energy (Eg) < the photon energy (hv)

Photodiodes

The photocurrent is proportional to the optical power $I_p = R_d P_{in}$

The constant R_d is the ***responsivity***

$$R_d = \eta \frac{q}{h\nu} = \eta \frac{\lambda}{1.24} \text{ [A/W] with } \lambda \text{ in } \mu\text{m}$$

- η = the ***quantum efficiency*** = the number of e–h pairs per incident photon
- Ideally η = 1

R_d increases with λ until $h\nu = E_g$

- $R_d \rightarrow 0$ when the photon energy becomes too low

Si or GaAs can be used for short wavelengths (λ < 900 nm)

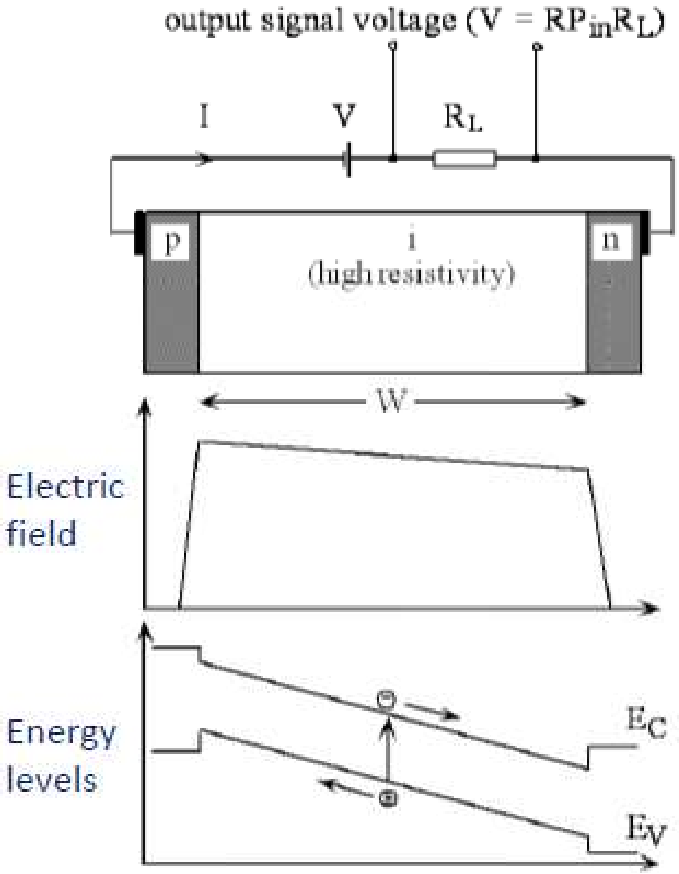
InGaAs is most common at 1.3 and 1.55 µm

Most communication systems use reverse-biased p–n junctions (photodiodes) of two main types:

- p–i–n photodiodes
- Avalanche photodiodes (APD)

p–i–n diodes

absorption of photons ⇒ e–h pair generation ⇒ carrier drift due to built-in and applied field ⇒ induced current in the external circuit



- p–i–n diode: p–n junction with an intrinsic (un-doped) layer
- Response time is limited by the transit time through the i-region

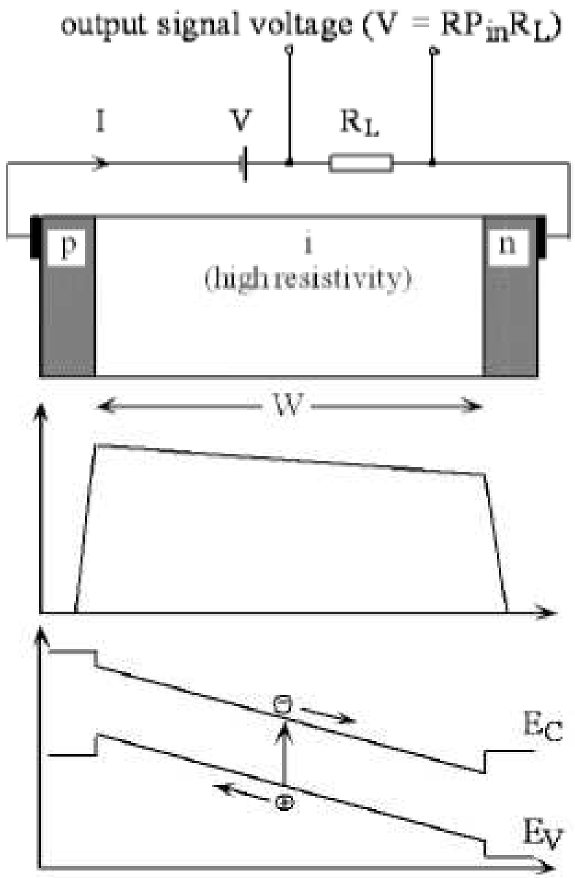
$$\tau_{tr} = \frac{W}{v_s}$$

- Responsivity increases with W ⇒ a trade-off between responsivity and speed
- High speed (~50 GHz) diodes with η close to unity are available

p–i–n diodes, performances

p–n diodes are limited by diffusion (absorption outside the depletion region)

In a p–i–n diode, the depletion region is wide (intrinsic, undoped)

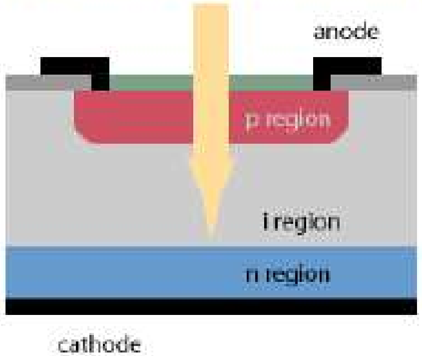


- p–i–n bandwidth limitations:
  - Parasitic capacitance
    - Reduces the speed of voltage changes
  - Transit time
    - Takes time to collect the carriers
- The *dark current* should be low
  - Current without input signal
    - Due to stray light and thermal generation of carriers

Examples of p–i–n diodes

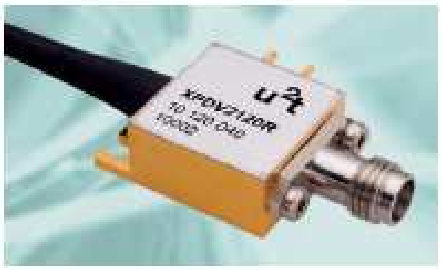
Schematic picture of a p–i–n diode

Green is anti-reflection coating



p–i–n diodes without

and with "pigtail"



- Important parameters are:
  - Bandwidth
  - Sensitivity
  - Responsivity
  - Polarization dependence
    - No dependence is preferred

Photodetector Avalanche photodiodes (APDs)

An APD is a p–i–n diode with an extra layer next to the i-region

- Gives gain through ***impact ionization*** and amplifies the signal
- The responsivity can be >> $q/h\nu$

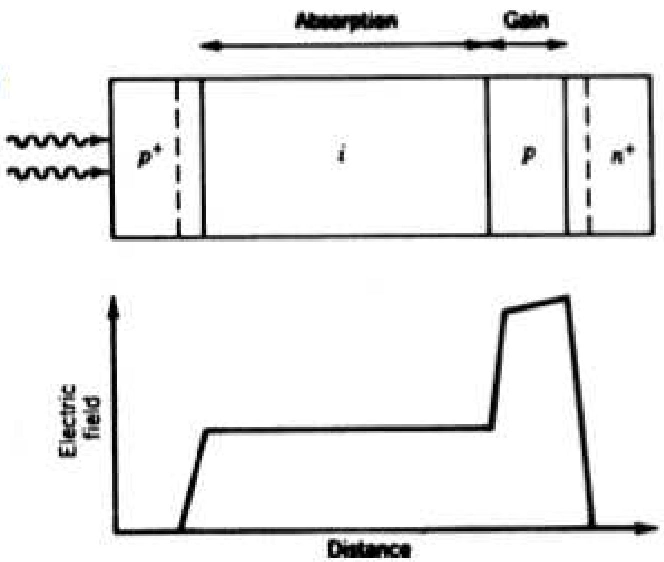
The responsivity of an APD is

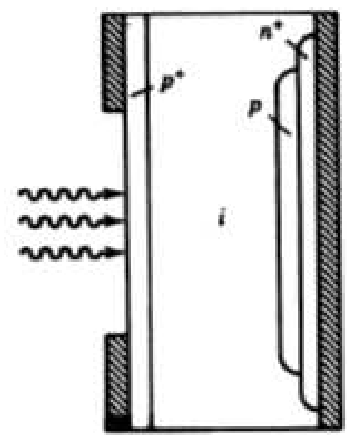
- *M* is the ***multiplication factor***

$$R_{\mathrm{APD}} = M\frac{\eta q}{h\nu} = MR_d$$

The increased responsivity comes at the expense of

- Enhanced noise
- Reduced bandwidth





APD multiplication factor

The multiplication factor *M* depends on the geometry of the APD, the electric field etc

The frequency dependence is

$$M(\omega) = \frac{M(0)}{\sqrt{1 + [\omega \tau_e M(0)]^2}}$$

- τ_e is the effective transit time for the multiplication process
- A trade-off between multiplication and bandwidth

Si-APDs have very good performance

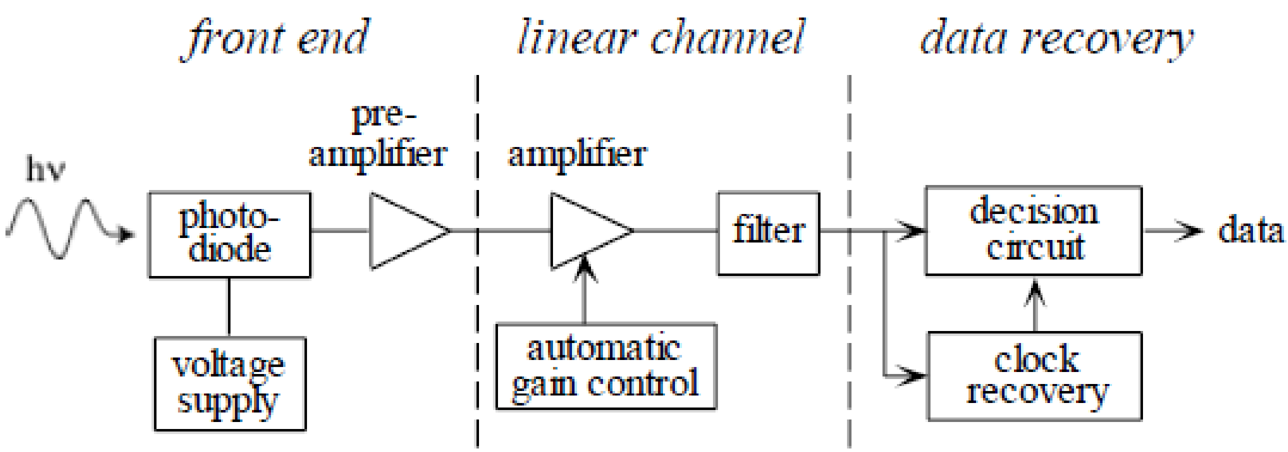
- $M > 100$, high bandwidth, relatively low noise
- Very useful for systems operating near 0.8 µm

InGaAs-APDs can be used at 1.3 and 1.55 µm

- Suffer from smaller multiplication and bandwidth, and higher noise

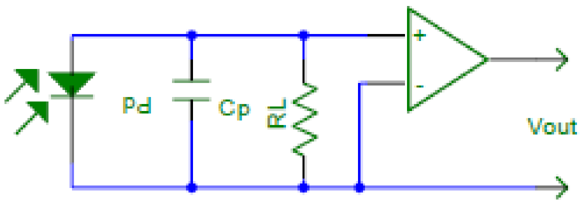
Receiver design

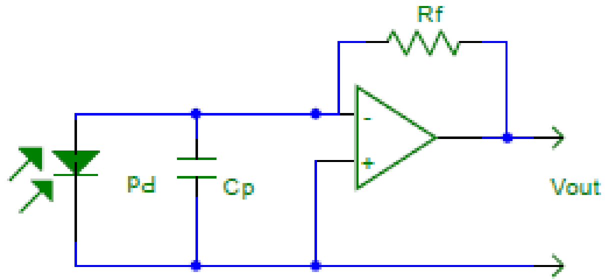
- The digital receiver consists of three parts:
  - Front end (photo-detector, trans-impedance amplifier)
  - Linear channel (amplifier, low-pass filter)
  - Data recovery (clock recovery, decision circuit)



Receiver front-ends

Transimpedance front-end





- Simple
- Electrically stable
- Low sensitivity for small R_L
- Small bandwidth for high R_L

$$\Delta f = \frac{1}{2\pi R_L C_p}$$

- High bandwidth
- High sensitivity
- Potentially unstable

$$\Delta f = \frac{G}{2\pi R_f C_p}$$

Effective input resistance = R_f/G

Linear channel

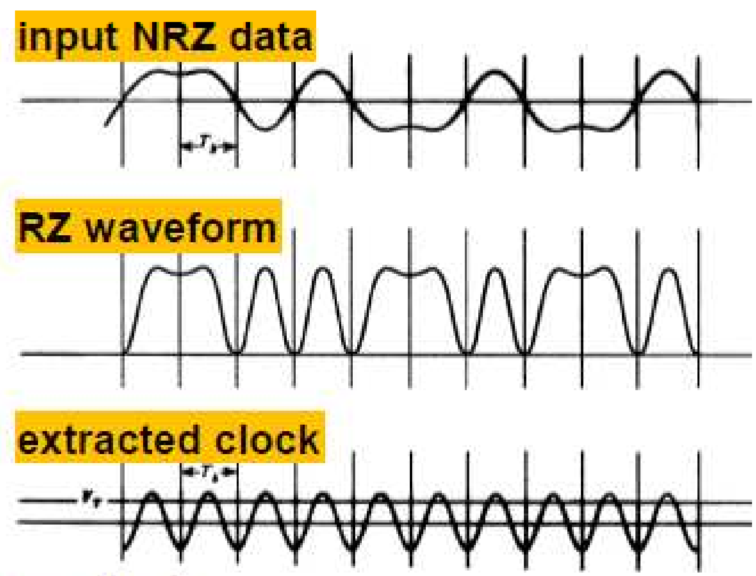
- The linear channel consists of:
  - A high-gain amplifier with automatic gain control
    - Const. average output voltage irrespective of the input (within limits)
  - A low-pass filter with bandwidth chosen to:
    - Reject noise outside signal bandwidth
    - Avoid introducing inter-symbol-interference (ISI)
- The best situation is when the filter (and not other components) limits
- the overall bandwidth of the receiver
- The output voltage spectrum is given by $H_{out}(\omega) = H_T(\omega)H_p(\omega)$
  - $H_p(\omega)$ - photocurrent spectrum
  - $H_T(\omega)$ - total transfer funct of the front end and the linear channel
- Normally, $H_T(\omega)$ is dominated by the filter transfer function
  - $HT(\omega) \approx Hf(\omega)$

Optical receivers

Data recovery

The data-recovery section consists of

- A ***clock-recovery circuit***
  - Extracting a sinusoidal component at $f = B$ to enable proper synchronization of the decision circuit
- Easily done for an OOK RZ signal with a narrow-band filter
  - The signal contains a delta function at $f = B$
- More difficult for NRZ
  - No sinusoidal spectral components are present
  - Can use a full-wave rectifier to convert the NRZ signal to RZ containing a delta function at $f = B$
- A ***decision circuit*** comparing the input voltage with a threshold at the time obtained from the clock recovery
  - Deciding whether a "1" or a "0" was received



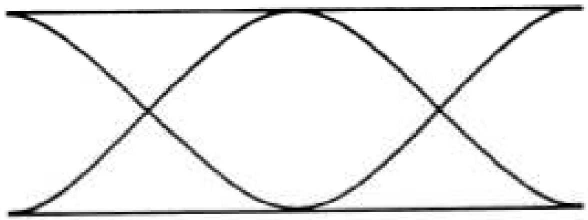
Eye diagrams

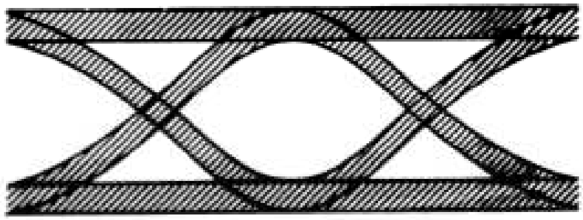
The ***eye diagram*** is a superposition of all bits on top of each other

- Looks like an eye
- Gives a visual way to monitor the receiver performance

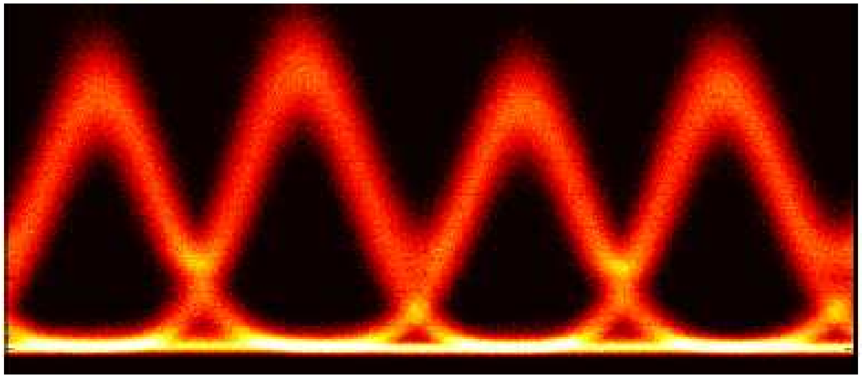
Left: An ideal NRZ eye diagram

Right: An eye diagram degraded by noise and timing jitter

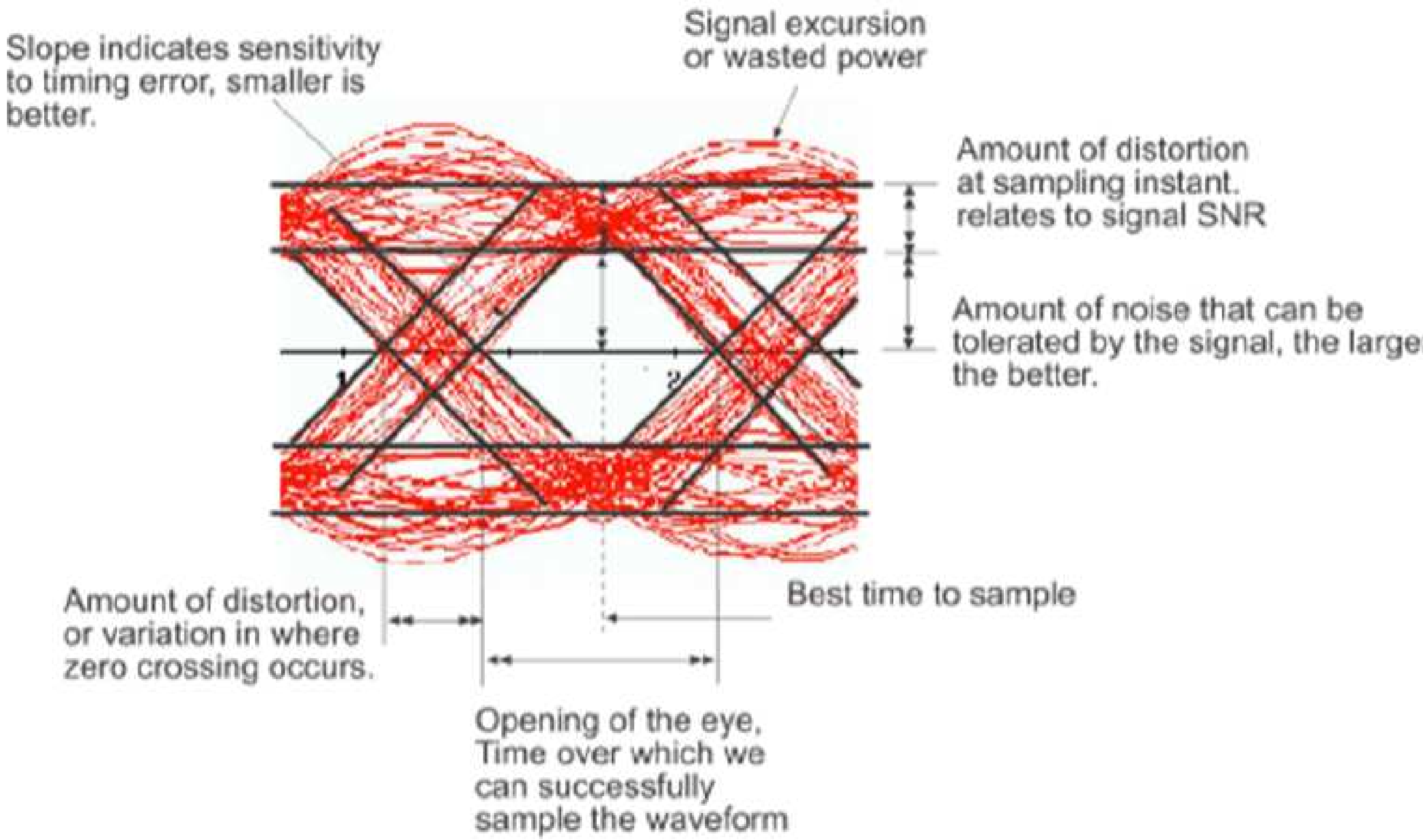




A measured RZ eye diagram at 640 Gbit/s



Eye diagram interpretation



Receiver noise

- The detected photo current in the receiver will contain noise
- There are two fundamental sources of noise
  - ***Shot noise*** due to field and charge quantization
  - ***Thermal noise*** due to thermal motion of charges
- The total current, signal + noise, can be written

$$I(t) = R_d P_{in}(t) + i_s(t) + i_T(t)$$

- In addition, there can also be optical noise in Pin
  - Comes from lasers and optical amplifiers
  - Will be treated later in the course
- Remember

$$I_p(t) = R_d P_{in}(t)$$

Shot noise

Shot noise arises from the particle nature of the photocurrent

- Current consists of electrons that can only be described statistically
- Current is not constant but fluctuates
- Compare with cars on a highway or hails on a roof

The variance of the shot noise photocurrent is

$$\sigma_s^2 = \langle i_s^2(t) \rangle = 2\int_0^{\Delta f} S_s(f)\,df = 2qI_p\Delta f$$

- Δf is the effective noise bandwidth of the receiver
- $S_s(f)$ is the shot noise two-sided ***power spectral density*** (PSD)

If the detector dark current I_d cannot be neglected we have

$$\sigma_s^2 = 2q(I_p + I_d)\Delta f$$

- Originating from stray light or thermally generated e–h pairs

Thermal noise

Thermal noise originates from the thermal motion of the electrons

The two-sided PSD is

$$S_T(f) = \frac{2hf}{R_L\left[\exp(hf/k_B T) - 1\right]} \approx \frac{2k_B T}{R_L}$$

- k_B is Boltzmann's constant
- T is the temperature
- R_L is the load resistance

The noise variance is

$$\sigma_T^2 = \left\langle i_T^2(t) \right\rangle = 2\int_0^{\Delta f} S_T(f)\,df \approx (4k_B T / R_L)\Delta f$$

In addition, thermal noise is also generated in electrical amplifiers

- Introduce the ***amplifier noise figure*** F_n to obtain

$$\sigma_T^2 = (4k_B T / R_L) F_n \Delta f$$

Photodetec Signal-to-noise ratio (SNR)

The different noise sources are uncorrelated

- We obtain the total noise power according to

$$\sigma^2 = \left\langle (\Delta I)^2 \right\rangle = \sigma_s^2 + \sigma_T^2 = 2q(I_p + I_d)\Delta f + (4k_B T / R_L)F_n \Delta f$$

The ***signal-to-noise ratio*** (SNR) of an electrical signal is defined as

$$\boxed{\mathrm{SNR} = \frac{\text{average signal power}}{\text{noise power}} = \frac{{I_p}^2}{\sigma^2}}$$

This definition is for an analog signal

- This is not the usual meaning of "SNR" in digital communication theory
  - Instead E_b/N_0 or E_s/N_0 is used there
    - E_b is the energy per bit
    - E_s is the energy per symbol
    - N_0 is the noise PSD

Noise in p–i–n receivers

For a p–i–n receiver we have

$$\mathrm{SNR} = \frac{R_d^2 P_{in}^2}{2q(R_d P_{in} + I_d)\Delta f + 4(k_B T / R_L) F_n \Delta f}$$

When thermal noise dominates, we have

$$\mathrm{SNR} = \frac{R_L R_d^2 P_{in}^2}{4 k_B T F_n \Delta f} \propto P_{in}^2$$

When shot noise dominates, we have

$$\mathrm{SNR} = \frac{R_d P_{in}}{2q\Delta f} = \frac{\eta P_{in}}{2h\nu_0 \Delta f} \propto P_{in}$$

We note:

- Different scaling with input power in the two limits
- Thermal noise dominates at low input power
- Shot noise dominates at high input power

Noise in APD receivers

Since $R_{APD} = M R_d$, the power of the current increases by M^2

- But the noise increases too, so the SNR increase is smaller

The APD shot noise variance is

$$\sigma_s^2 = 2qM^2 F_A (R_d P_{in} + I_d)\Delta f$$

The ***excess noise factor*** is

$$F_A(M) = k_A M + (1 - k_A)(2 - 1/M)$$

- $1 < F_A < M$ since $0 < k_A < 1$, ($k_A = \alpha_h/\alpha_e$, see (4.2.3))

The SNR becomes

$$\boxed{\mathrm{SNR} = \frac{(MR_d P_{in})^2}{2qM^2 F_A (R_d P_{in} + I_d)\Delta f + 4(k_B T / R_L) F_n \Delta f}}$$

- The shot-noise is increased by $M^2 F_A$

Noise in APD receivers

In the thermal noise limit we have

$$\mathrm{SNR} = \frac{R_L R_d^2 M^2 P_{in}^2}{4k_B T F_n \Delta f} \propto M^2 P_{in}^2$$

– A factor of M^2 higher than for the p–i–n

In the shot noise limit we have

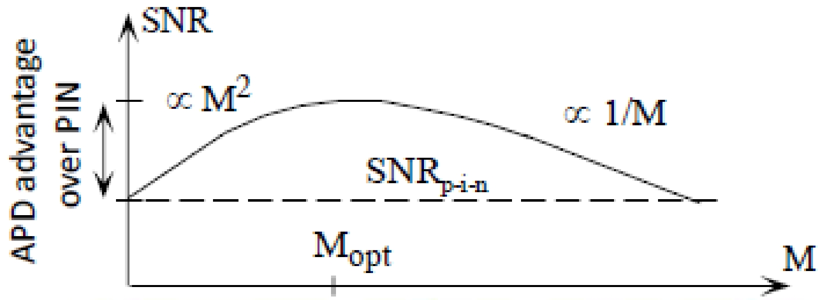
$$\mathrm{SNR} = \frac{R_d P_{in}}{2qF_A \Delta f} = \frac{\eta P_{in}}{2h\nu_0 F_A \Delta f} \propto \frac{P_{in}}{F_A}$$

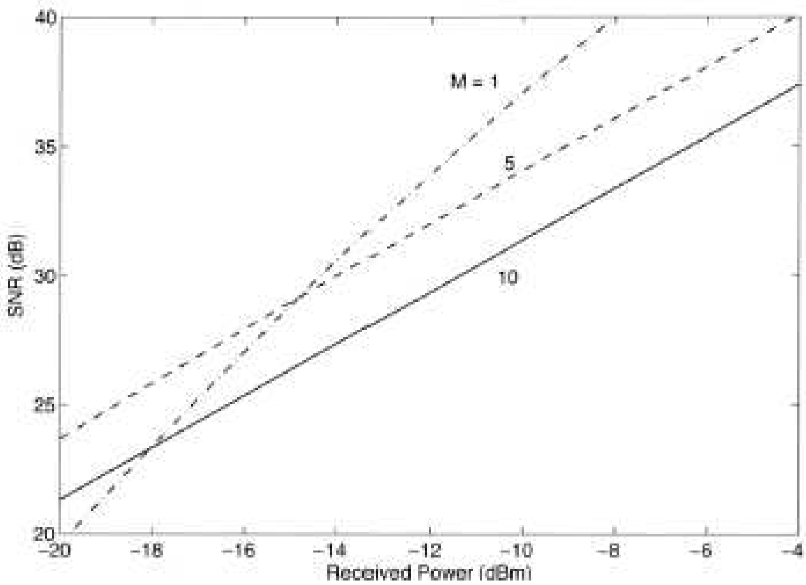
– A factor of F_A lower than for the p–i–n diode

- The SNR is increased by an APD in the thermal-noise limit
- The SNR is decreased by an APD in the shot-noise limit

The APD vs the p–i–n

- The SNR (Δf = 30 GHz) for a p–i–n receiver and an APD receiver
  - APD is best at low power
  - p–i–n is best at high power
  - M = 10 is worse than M = 5





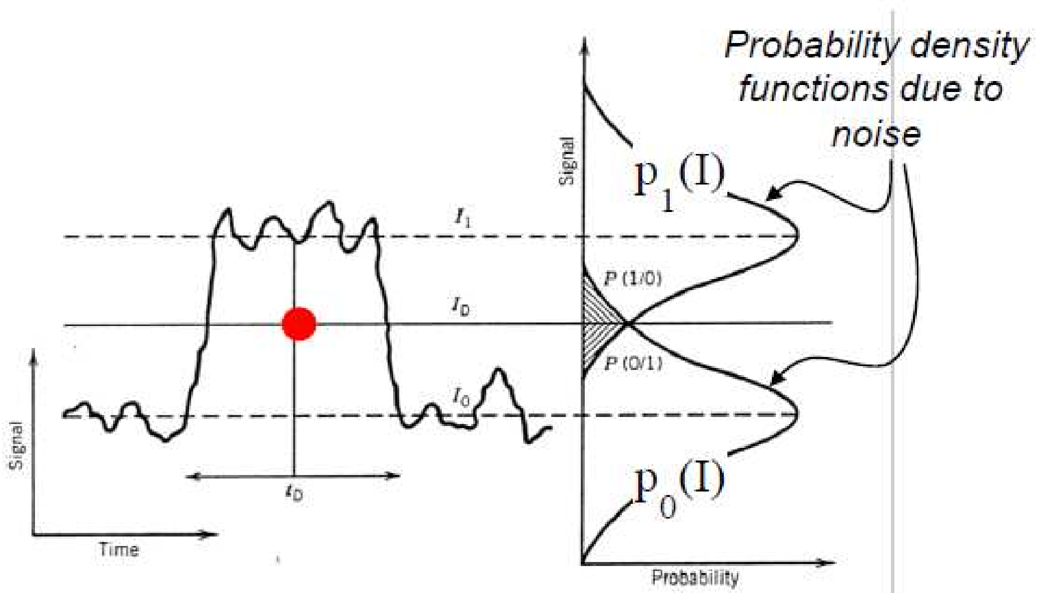
- There is an optimum value for M

$$M_{opt} \approx \left[\frac{4k_B T F_n}{k_A q R_L (R_d P_{in} + I_d)} \right]^{1/3}$$

- Optimum value depends on $k_A = \alpha_h/\alpha_e$
  - Highest M_{opt}~100 for silicon APD
  - Highest M_{opt}~10 for InGaAs APD

Bit error rate

- The ***bit error rate*** (BER) is the probability that a bit is incorrectly identified by the receiver (due to the noise and other signal distortion)
  - A better name would be ***bit error probability***
  - A traditional requirement for optical receivers is BER < 10^{-9}
- The ***receiver sensitivity*** is the minimum averaged received optical power required to achieve the target BER
- Figure shows:
  - A signal affected by noise
  - The PDFs for the upper and lower current levels
  - The decision threshold I_D
  - The dashed area indicates errors



BER calculation

- $p(1)$ is the probability to send a "one"
- $P(0|1)$ is the probability to detect a sent out "one" as a "zero"

$$\mathrm{BER} = p(1)P(0\,|\,1) + p(0)P(1\,|\,0) = \{p(1) = p(0) = 1/2\} = \tfrac{1}{2}\left[P(0\,|\,1) + P(1\,|\,0)\right]$$

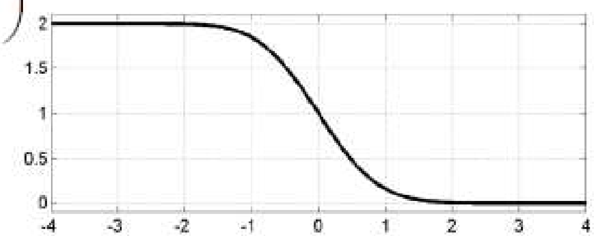
- Assume that the noise has Gaussian statistics
  - I_1 (I_0) is the upper (lower) current level
  - σ_1 (σ_0) is the standard deviation of the upper (lower) level

$$P(0\,|\,1) = \frac{1}{\sigma_1\sqrt{2\pi}} \int_{-\infty}^{I_D} \exp\left(-\frac{(I - I_1)^2}{2\sigma_1^2}\right) dI = \frac{1}{2}\operatorname{erfc}\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right)$$

$$P(1\,|\,0) = \frac{1}{\sigma_0\sqrt{2\pi}} \int_{I_D}^{\infty} \exp\left(-\frac{(I - I_0)^2}{2\sigma_0^2}\right) dI = \frac{1}{2}\operatorname{erfc}\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right)$$

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} \exp(-y^2)\,dy$$

The erfc function



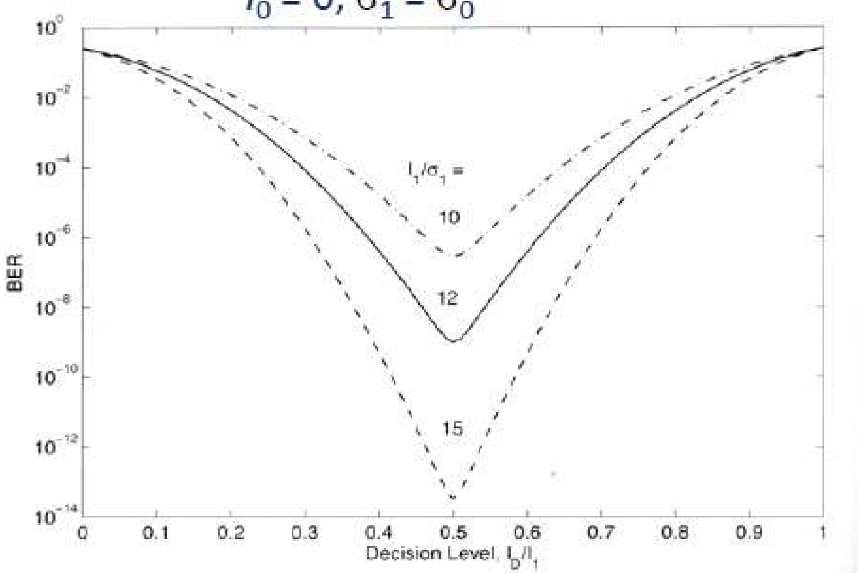
BER calculation

These expressions give us the BER

$$\mathrm{BER} = \frac{1}{4}\left[\mathrm{erfc}\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right) + \mathrm{erfc}\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right)\right]$$

BER using assumptions

$I_0 = 0$, $\sigma_1 = \sigma_0$



- BER depends on I_D
- Note: In general σ_1 and σ_0 are not equal
- Example: Shot noise depends on the current $\Rightarrow \sigma_1 > \sigma_0$ since $I_1 > I_0$

Optimal decision threshold

Minimize the BER using $d(\mathrm{BER})/dI_D = 0$

- Optimal value is the intersection of the PDF for the "one" and "zero" levels

Exact expression is given in the book

Choosing I_D according to expression below is a good approximation

$$(I_D - I_0)/\sigma_0 = (I_1 - I_D)/\sigma_1 \equiv Q \qquad I_D = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}$$

Notice the definition of Q

- Often used as a measure of signal quality

Thermal case: $\sigma_1 = \sigma_0$ and $I_D = (I_1 + I_0)/2$

When shot noise cannot be neglected, I_D shifts towards the "zero" level

The Q value

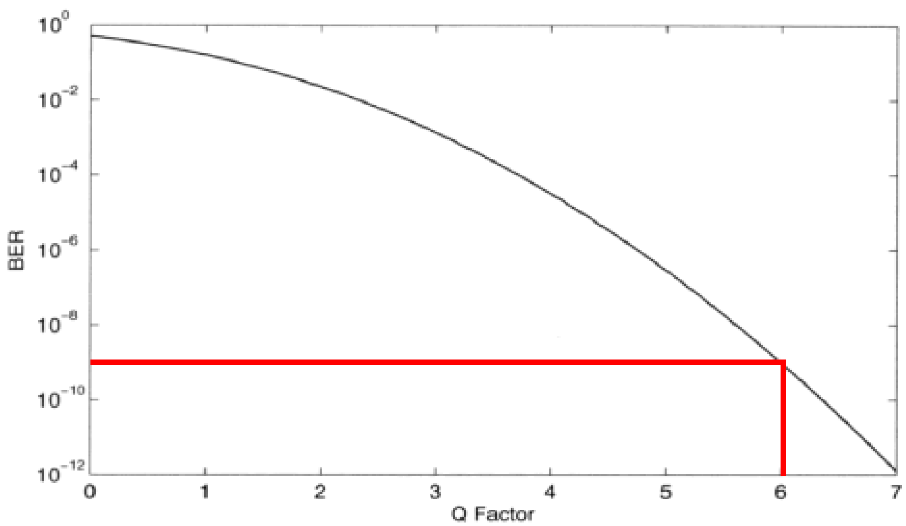
The Q value is a measure of the ***eye opening*** since

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$$

The ***optimum BER*** is related to the *Q* value as

$$\text{BER} = \frac{1}{2}\text{erfc}\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{\exp(-Q^2/2)}{Q\sqrt{2\pi}}$$

– If currents and noise levels are known, the BER can be found from *Q*



Q is often defined in dB scale as

$$Q^2(\text{in dB}) = 20\log_{10} Q$$

– Example: BER = 10^{-9} corresponds to *Q* = 6 or 15.6 dB

Minimum average received power

Consider the following case:

- NRZ data in which "zero" bits contain no optical power, neglect dark current
- The receiver uses an APD, the p–i–n case is obtained by setting $M = F_A = 1$

The average current for a "one" is $I_1 = MR_d P_1 = 2MR_d \bar{P}_{\text{rec}}$

where the average received power is $\bar{P}_{\text{rec}} = (P_1 + P_0)/2 = P_1/2$

The Q value is

$$Q = \frac{I_1}{\sigma_1 + \sigma_0} = \frac{2MR_d \bar{P}_{\text{rec}}}{(\sigma_s^2 + \sigma_T^2)^{1/2} + \sigma_T}$$

where the shot noise is $\sigma_s^2 = 2qM^2 F_A R_d (2\bar{P}_{\text{rec}})\Delta f$

and the thermal noise is $\sigma_T^2 = (4k_B T / R_L) F_n \Delta f$

The receiver sensitivity is then

$$\bar{P}_{\text{rec}} = \frac{Q}{R_d}\left(qF_A Q\Delta f + \frac{\sigma_T}{M}\right)$$

Minimum average received power

When thermal noise dominates in a p–i–n receiver, we have

$$(\bar{P}_{\text{rec}})_{\text{pin}} \approx Q\sigma_T / R_d \propto \sqrt{\Delta f}$$

- This corresponds to $\text{SNR} = I_1^2 / \sigma_1^2 = 4Q^2$
- Example: $Q = 6$, $R_d = 1$ A/W, $\sigma_T = 0.1$ µA ⇒
  $P_{rec} = 0.6$ µW, SNR = 144 = 21.6 dB

When shot noise dominates in a p–i–n receiver, we have

$$(\bar{P}_{\text{rec}})_{\text{ideal}} = (q\Delta f / R_d)Q^2 \propto \Delta f$$

- This corresponds to $\text{SNR} = I_1^2 / \sigma_1^2 = Q^2$
- Example: $Q = 6$ ⇒ SNR = 36 = 15.6 dB

Optimum sensitivity in APD receivers

In a receiver dominated by thermal noise, an APD will increase the SNR

There is an optimum gain, given by

$$M_{\text{opt}} = k_A^{-1/2}\left(\frac{\sigma_T}{Qq\Delta f} + k_A - 1\right)^{1/2} \approx \left(\frac{\sigma_T}{k_A Qq\Delta f}\right)^{1/2}$$

The corresponding sensitivity is

$$(\bar{P}_{\text{rec}})_{\text{APD}} = (2q\Delta f / R_d)Q^2(k_A M_{\text{opt}} + 1 - k_A)$$

Note: $P_{\text{rec}} \sim \Delta f$ and not $\sim\sqrt{\Delta f}$ as for thermally limited receivers

For InGaAs APDs, the sensitivity is typically improved over a p–i–n diode receiver by 6–8 dB

Quantum limit of photo detection

At very low power levels, the noise statistics are no longer Gaussian

Denote the average number of photons per "one" bit by N_p

The probability of generating m electron-hole pairs is then given by the ***Poisson distribution***

$$P_m = \exp(-N_p) N_p^m / m!$$

Assume: No thermal noise, $P_0 = 0$, threshold is at one detected photon

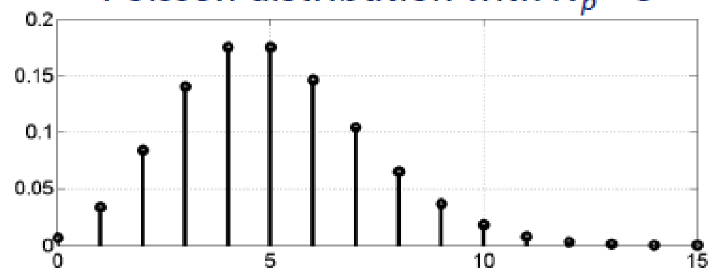
$$\mathrm{BER} = \tfrac{1}{2}\left[P(0\,|\,1) + P(1\,|\,0)\right] = \tfrac{1}{2}\left[p(m=0) + 0\right] = \tfrac{1}{2}\exp(-N_p)$$

For BER < 10^{-9}, we must have $N_p > 20$ photons per "one" bit

This corresponds to a power in a "one" of $P_1 = N_p h\nu B$ and an average received power $P_{rec} = N_p h\nu B/2$

Example: B = 10 Gbit/s, N_p=20 ⇒ P_{rec} = 13 nW at λ = 1550 nm

Poisson distribution with $N_p = 5$



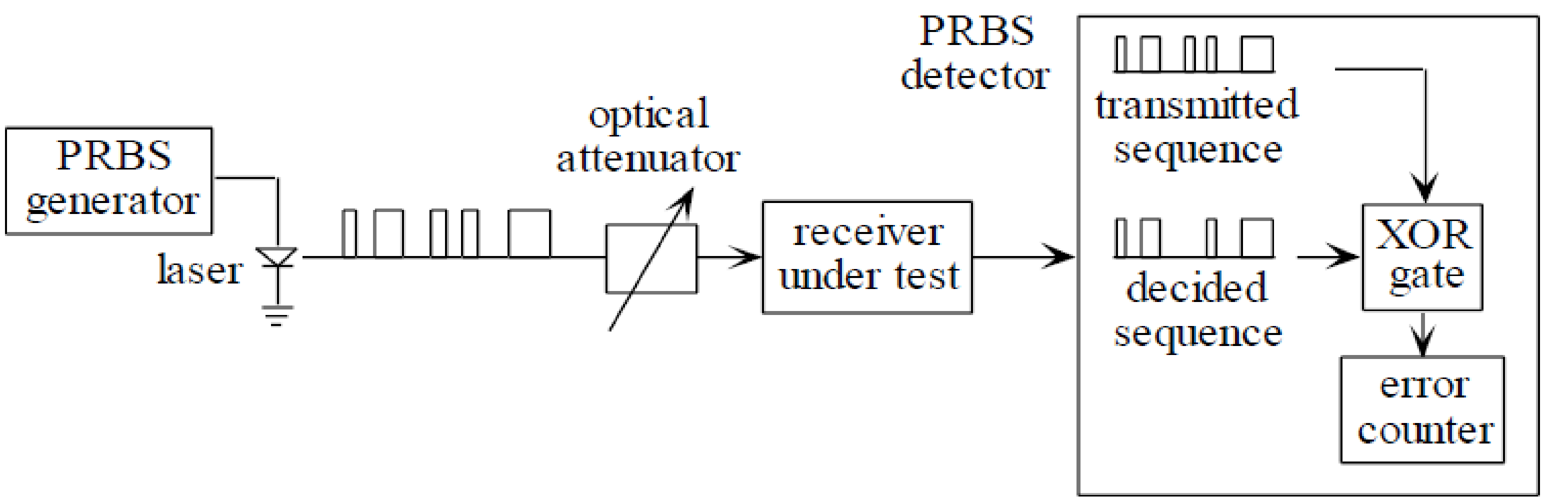
Receiver characterization

Receivers are experimentally studied using a long ***pseudorandom binary sequence*** (PRBS)

- Random data is hard to generate
- Random data is not periodic
- Typical length $2^{15}-1$

The BER is measured as a function of received average optical power

- Sensitivity = average power corresponding to a given BER (often 10^{-9})



Sensitivity degradation

So far, we have discussed an ideal situation

- Perfect pulses corrupted only by (inevitable) noise

In reality, the receiver sensitivity is degraded

- There are additional sources of signal distortion

The corresponding necessary increase in average received power to achieve a certain BER is called the ***power penalty***

Also without propagation in a fiber, a power penalty can arise

Examples of degrading phenomena include:

- Limited modulator extinction ratio
- Transmitter intensity noise
- Timing jitter

Extinction ratio (ER)

The ***extinction ratio*** (ER) is defined as $r_{ex} = P_0/P_1$

- P_0 (P_1) is the emitted power in the off (on) state
- Ideally, $r_{ex} = 0$

Different for direct and external modulation

We use that

- The average received power is $P_{rec} = (P_1 + P_0)/2$
- The definition of the Q-parameter is $Q = (I_1 - I_0)/(\sigma_1 + \sigma_0)$

We find the sensitivity degradation to be

$$Q = \left(\frac{1 - r_{ex}}{1 + r_{ex}}\right)\frac{2R_d\bar{P}_{rec}}{\sigma_1 + \sigma_0}$$

Extinction ratio (ER), power penalty

If thermal noise dominates, then $\sigma_1 = \sigma_0 = \sigma_T$, and the sensitivity is

$$\bar{P}_{rec}(r_{ex}) = \left(\frac{1+r_{ex}}{1-r_{ex}}\right)\frac{\sigma_T Q}{R_d}$$

The power penalty is (in dB)

$$\delta_{ex} = 10\log_{10}\left(\frac{\bar{P}_{rec}(r_{ex})}{\bar{P}_{rec}(0)}\right) = 10\log_{10}\left(\frac{1+r_{ex}}{1-r_{ex}}\right)$$

Laser biased below threshold

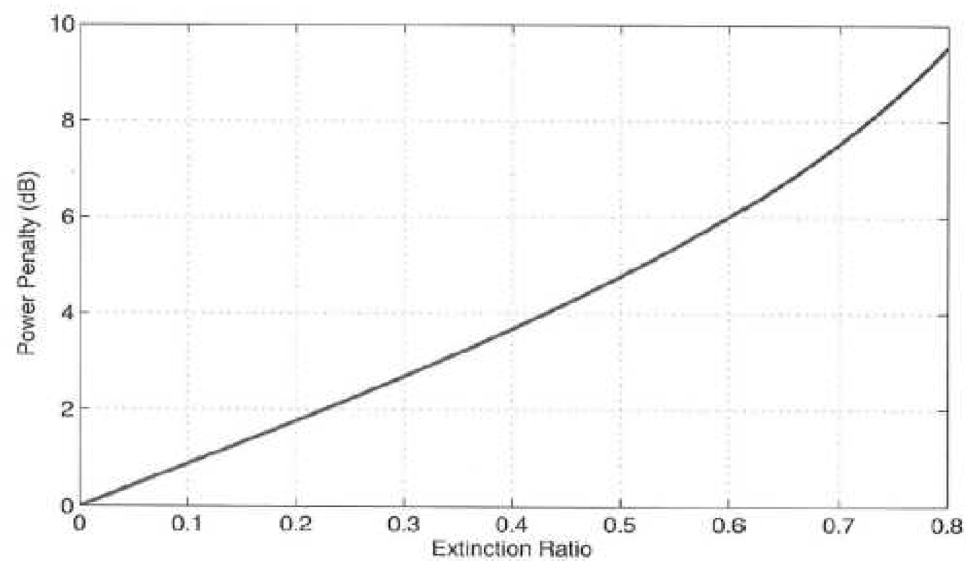
$r_{ex} < 0.05$ (–13 dB) $\Rightarrow \delta_{ex} < 0.4$ dB

For a laser biased above threshold

$r_{ex} > 0.2 \Rightarrow \delta_{ex} > 1.5$ dB

The penalty is independent of Q and BER

The penalty for APD receivers is larger than for p–i–n receivers



Intensity noise (RIN)

Intensity noise in LEDs and semiconductor lasers add to the thermal and shot noise

Approximately, this is included by writing

$$\sigma^2 = \sigma_s^2 + \sigma_T^2 + \sigma_I^2$$

where

$$\sigma_I = R_d \left\langle \Delta P_{\text{in}}^2 \right\rangle^{1/2} = R_d P_{\text{in}} r_I \qquad r_I^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{RIN}(\omega) d\omega$$

(The RIN spectrum was discussed earlier)

The parameter r_I is the inverse SNR of the transmitter

Assuming zero extinction ratio and using that

$$\sigma_s = (4qR_d \bar{P}_{\text{rec}} \Delta f)^{1/2} \qquad \sigma_I = 2r_I R_d \bar{P}_{\text{rec}}$$

we can now write the Q-value as

$$\boxed{Q = \frac{2R_d \bar{P}_{\text{rec}}}{(\sigma_s^2 + \sigma_T^2 + \sigma_I^2)^{1/2} + \sigma_T}}$$

Intensity noise (RIN), power penalty

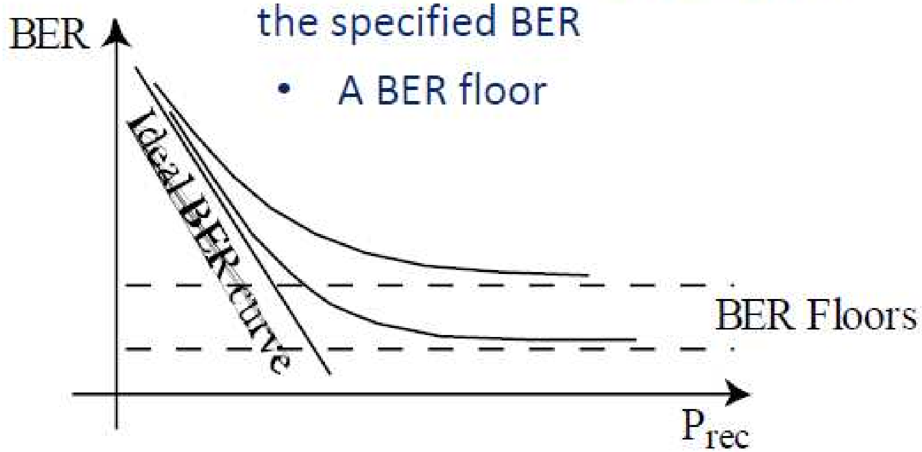
- The receiver sensitivity is found to be

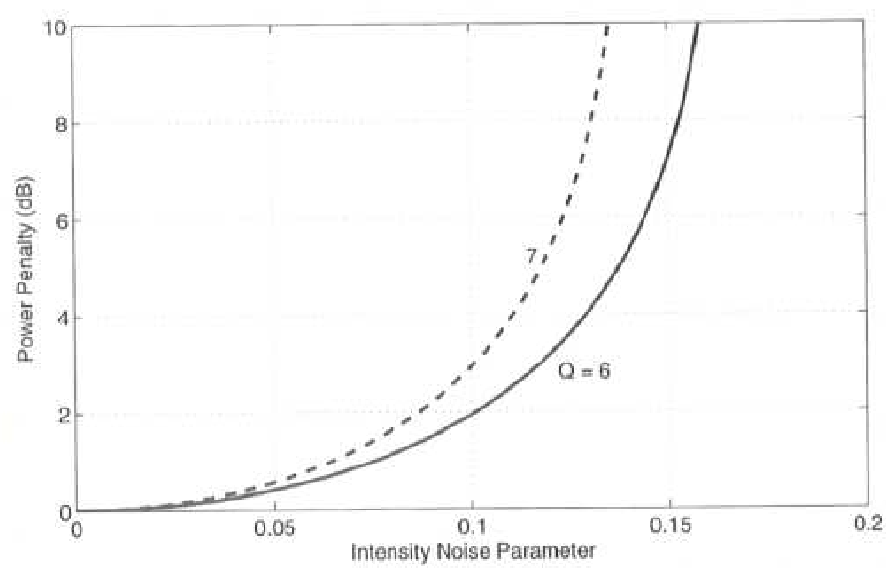
$$\bar{P}_{\text{rec}}(r_I) = \frac{Q\sigma_T + Q^2 q\Delta f}{R_d(1 - r_I^2 Q^2)}$$

- The power penalty is

$$\delta_I = 10\log_{10}\left[\bar{P}_{\text{rec}}(r_I)/\bar{P}_{\text{rec}}(0)\right] = -10\log_{10}(1 - r_I^2 Q^2)$$

- Note that $\delta_I \rightarrow \infty$ when $r_I \rightarrow 1/Q$
  - The receiver cannot operate at the specified BER
    - A BER floor





Timing jitter

The recovered clock is based on the received, noisy signal

- The decision time fluctuates and causes ***timing jitter***

The data is not sampled at the bit slot center

- Leads to additional fluctuations of the signal entering the decision circuit

In a thermally limited p–i–n receiver, we have

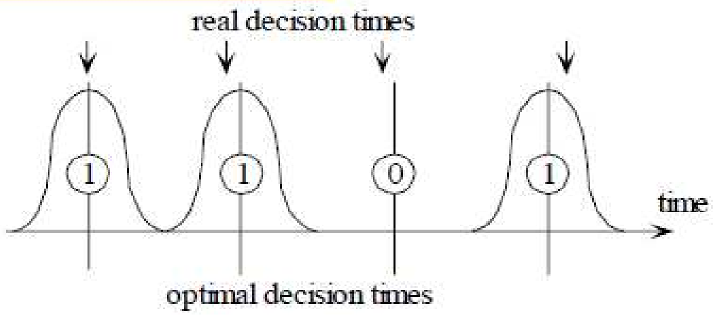
- Δi_j is the current fluctuation
- σ_j is the corresponding RMS value

$$Q = \frac{I_1 - \langle \Delta i_j \rangle}{(\sigma_T^2 + \sigma_j^2)^{1/2} + \sigma_T}$$

The penalty depends on the pulse shape, but for a "typical case"

$$\delta_j = 10\log_{10}\left(\frac{\overline{P}_{rec}(b)}{\overline{P}_{rec}(0)}\right) = 10\log_{10}\left(\frac{1-b/2}{(1-b/2)^2 - b^2Q^2/2}\right)$$

- $b = (4\pi^2/3 - 8)(B\tau_j)^2$
- τ_j is the RMS value of Δt



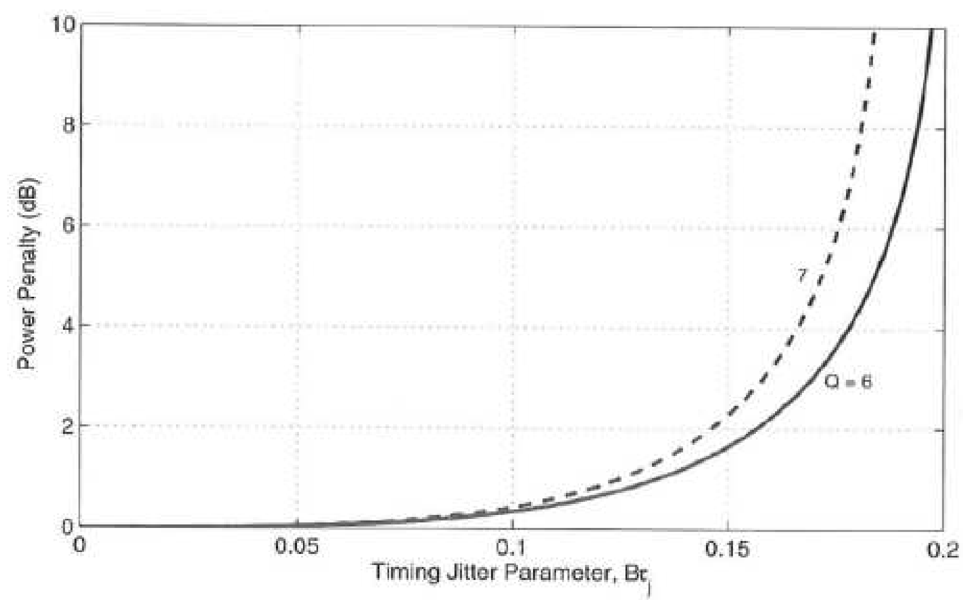
Timing jitter, power penalty

The power penalty depends on *Q* (BER)

– The penalty will be higher at a lower BER

Rule-of-thumb:

– The RMS value of the timing jitter should typically be smaller than 5–10% of the bit slot to avoid significant penalty



Receiver performance

Real sensitivities are

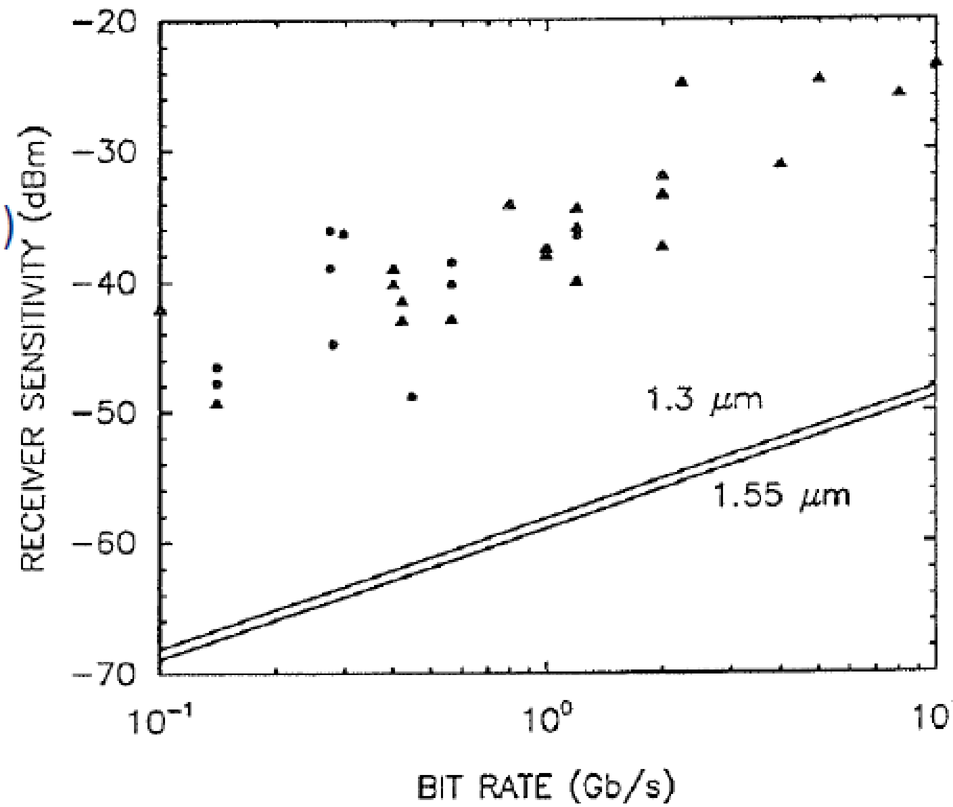
- ≈ 20 dB above the quantum limit for APDs
- ≈ 25 dB above the quantum limit for p–i–n diodes
- Mainly due to thermal noise

Figure shows

- Measured sensitivities for p–i–n diodes (circles) and APDs (triangles)
- Lines show the quantum limit

Two techniques to improve this

- Coherent detection
- Optical pre-amplification
- Both can reach sensitivities of only 5 dB above the quantum limit



Loss-limited lightwave systems

The maximum (unamplified) propagation distance is

$$L[\mathrm{km}] = \frac{10}{\alpha_f[\mathrm{dB/km}]}\log\left(\frac{P_{\mathrm{tr}}}{P_{\mathrm{rec}}}\right)$$

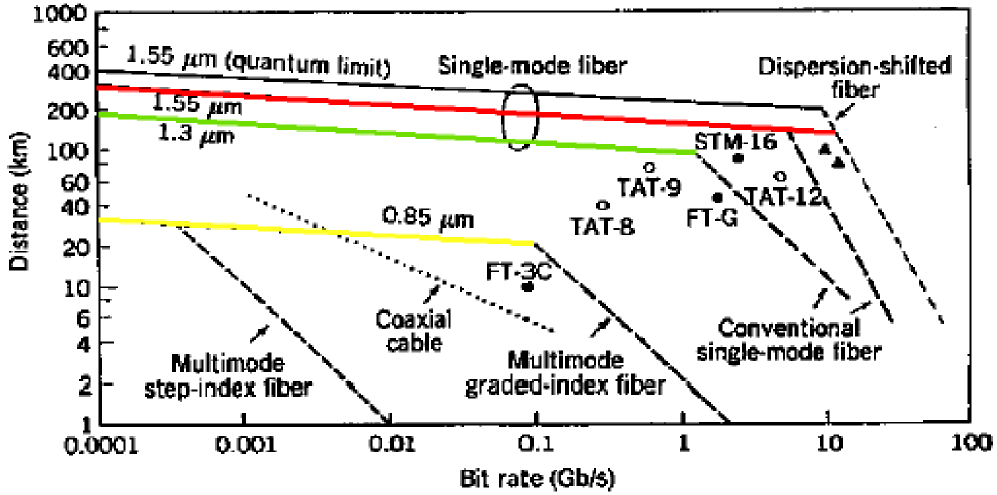
- P_{rec} is receiver sensitivity
- P_{tr} is transmitter average power
- α_f is the net loss of the fiber, splices, and connectors

P_{rec} and L are bit rate dependent

Table shows wavelengths with corresponding quantum limits and typical losses

λ [μm]	<Np>	α_f [dB/km]
0.85	300	2.5
1.3	500	0.4
1.55	500	0.2

- Loss-limited transmission
  - Transmitted power = 1 mW



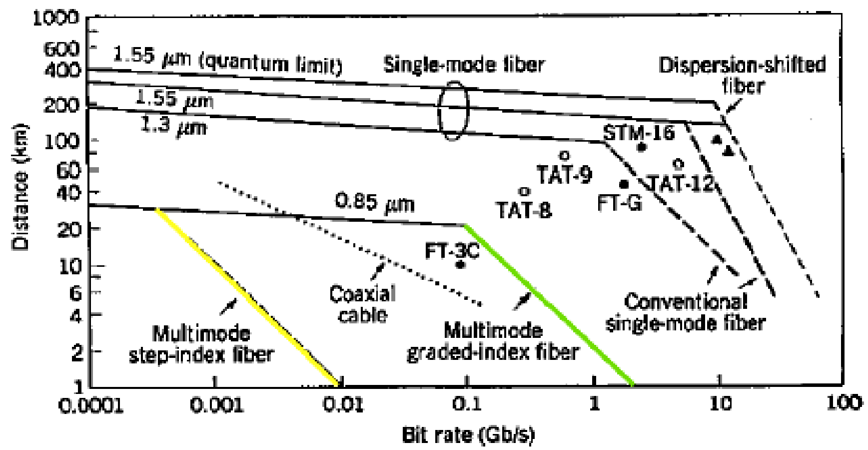
- λ = 850 nm, L_{max} = 10–30 km
- λ = 1.55 μm, L_{max} = 200–300 km

Dispersion-limited lightwave systems

Occurs when pulse broadening is more important than loss

The dispersion-limited distance depends on for example

- The operating wavelength
  - Since D is a function of λ
- The type of fiber
  - Multi-mode: step-index or graded-index
  - Single-mode: standard or dispersion-shifted
- Type of laser
  - Longitudinal multimode
  - Longitudinal singlemode
    - large or small chirp



- λ = 850 nm, multimode SI-fiber
  - Modal dispersion dominates
  - Disp.-limited for B > 0.3 Mbit/s

$$BL < c/(2n_1\Delta) \approx 10\,(\mathrm{Mbit/s}) \times \mathrm{km}$$

- λ = 850 nm, multimode GI-fiber
  - Modal dispersion dominates
  - Disp.-limited for B > 100 Mbit/s

$$BL < 2c/(n_1\Delta^2) \approx 2\,(\mathrm{Gbit/s}) \times \mathrm{km}$$

Dispersion-limited lightwave systems

λ = 1.3 µm, SM-fiber, MM-laser

- Material dispersion dominates
- Disp.-limited for *B* > 1 Gbit/s
- Using |*D*| σ_λ = 2 ps/nm

$$BL < 1/(4|D|\sigma_\lambda) \approx 125\,(\mathrm{Gbit/s}) \times \mathrm{km}$$

λ = 1.55 µm, SM-fiber, SM-laser

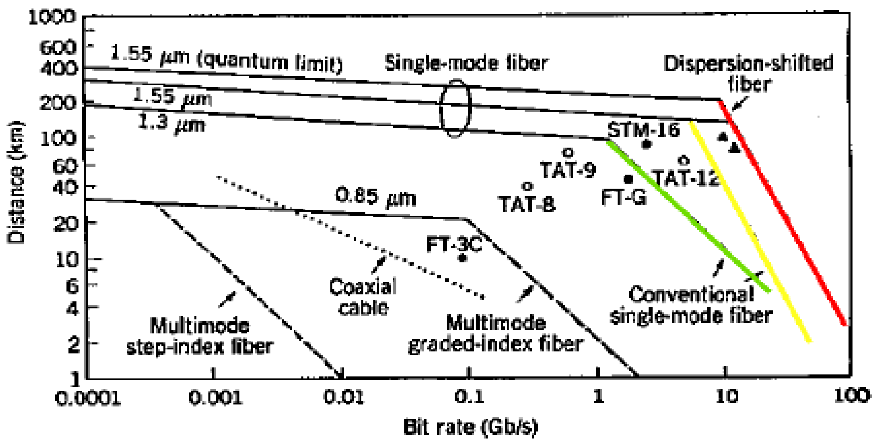
- Material dispersion dominates
- Using |*D*| = 16 ps/(nm×km)
- Disp.-limited for *B* > 5 Gbit/s

$$B^2L < 1/(16|\beta_2|) \approx 4000 (\mathrm{Gbit/s})^2 \times \mathrm{km}$$

λ = 1.55 µm, DS-fiber, SM-laser

- Material dispersion dominates
- Using |*D*| = 1.6 ps/(nm×km)
- Disp.-limited for *B* > 15 Gbit/s

$$B^2L < 1/(16|\beta_2|) \approx 40000 (\mathrm{Gbit/s})^2 \times \mathrm{km}$$



- Long systems often use in-line amplifiers
  - Loss is not a critical limitation
  - Dispersion must be compensated for
  - Noise and nonlinearities are important
  - PMD can be a problem

System design

Part of the system design is to make sure the BER demand can be met

- The ***power budget*** is a very useful tool
- The transmitter average power (P_{tr}) and the average power required at the receiver (P_{rec}) are often specified

$$\overline{P}_{\text{tr}}^{[\text{dBm}]} = \overline{P}_{\text{rec}}^{[\text{dBm}]} + C_L^{[\text{dB}]} + M_s^{[\text{dB}]} \qquad C_L^{[\text{dB}]} = \alpha_f^{[\text{dB/km}]} L + \alpha_{\text{con}}^{[\text{dB}]} + \alpha_{\text{splice}}^{[\text{dB}]}$$

- C_L is the total channel loss (sum of fiber, connector, and splice losses)
- M_s is the ***system margin*** (allowing penalties and degradation over time)
  - Typically M_s = 6–8 dB

A complete system is very complex and some of the parameters that must be considered are

- Modulation format, detection scheme, operating wavelength
- Transmitter and receiver implementation, type of fiber
- The trade-off between cost and performance
- The system reliability

Further sources of power penalty

The above mentioned power penalties were all due to the transmitter and the receiver

Several more sources of power penalty appear during propagation

- Modal noise (in multi-mode fibers)
- Mode-partition noise (in multi-mode lasers)
- Intersymbol interference (ISI) due to pulse broadening
- Frequency chirp
- Reflection feedback

All these involve dispersion

Power penalties in multi-mode fiber

Modal noise

- Different modes interfere over the fiber cross-section
  - Forms a time-varying "speckle" intensity pattern
  - The received power will fluctuate
- Problem occurs with highly coherent sources
- To avoid this
  - Use a single-mode fiber
  - Reduce coherence
    - Use a LED

Mode-partition noise

- The power in each longitudinal mode of a multimode laser varies with time
  - Output power is constant
- Different modes propagate at different velocities in a fiber
  - Additional signal fluctuation is caused and the SNR is degraded
- Negligible penalty if $BLD\sigma_{\lambda} < 0.1$

Power penalty due to pulse broadening

Broadening affects the receiver in two ways

- Energy spreads beyond the bit slot ⇒ ISI
- Pulse peak power is reduced for a given average received power
  - Reduces the SNR

Power penalty for Gaussian pulses assuming no ISI is

$$\delta_d = 10\log_{10}\left(\left|\frac{A(0)}{A(L)}\right|^2\right) = 10\log_{10}(\sigma/\sigma_0)$$

Assuming $\beta_3 \approx C \approx 0$ and a large source spectral width, we have

$$\frac{\sigma}{\sigma_0} = \sqrt{1+\left(\frac{LD\sigma_\lambda}{\sigma_0}\right)^2}$$

$$\delta_d = 5\log_{10}\left[1+(LD\sigma_\lambda/\sigma_0)^2\right]$$

Power penalty due to pulse broadening

Assuming $\beta_3 \approx C \approx 0$ and a small source spectral width, we have

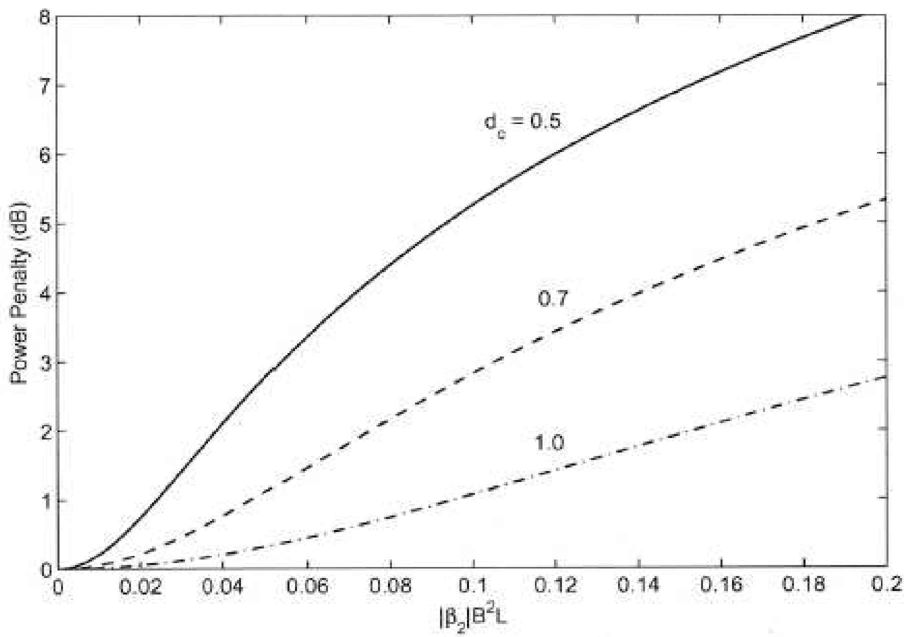
$$\delta_d = 5\log_{10}\left[1+\left(\frac{\beta_2 L}{2\sigma_0^2}\right)^2\right]$$

Agrawal introduces the ***duty cycle***

- A measure of the pulse width
- Defined as $d_c = 4\,\sigma_0/T_B$

The penalty depends on

- Dispersion parameter
- Fiber length
- Bit rate
- Pulse width (duty cycle)



Power penalty due to chirp

Frequency chirping increases the impact of dispersion

Occurs in directly modulated lasers

– Cannot modulate the amplitude without changing the phase

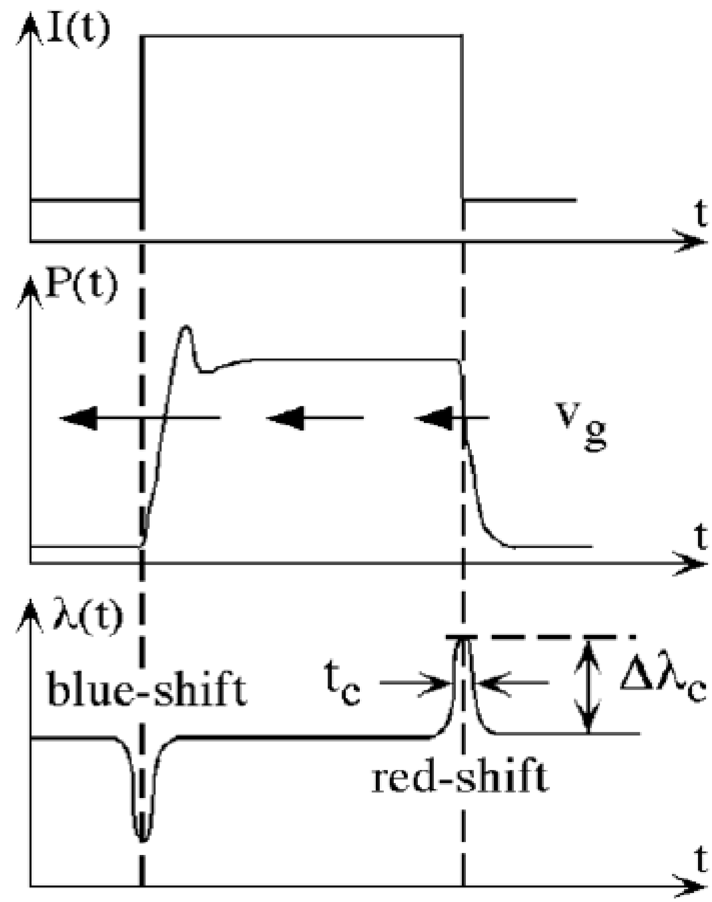
Figure shows driving current, output power and wavelength of a directly modulated laser

– $t_c \approx$ 100–200 ps = chirp duration

– $\Delta\lambda_c$ = spectral shift associated with the chirp

Exact impact is complicated

– Assume pulse is Gaussian with linear chirp



Power penalty due to chirp

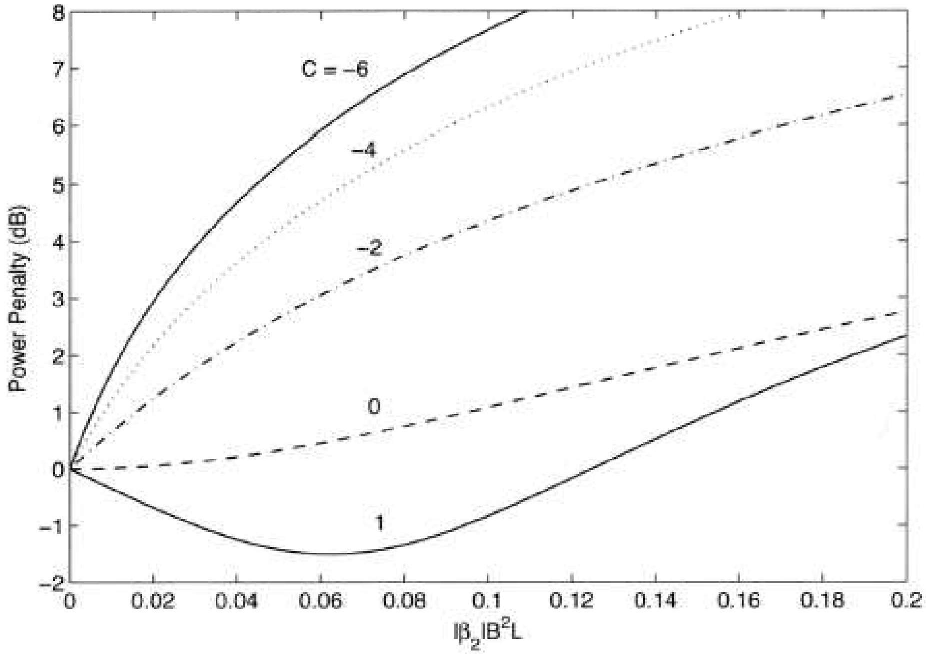
For chirped Gaussian pulses with $\beta_3 \approx 0$, we have

$$\delta_c = 5\log_{10}\left[\left(1+\frac{C\beta_2 L}{2\sigma_0^2}\right)^2 + \left(\frac{\beta_2 L}{2\sigma_0^2}\right)^2\right]$$

A chirp-free pulse ($C = 0$) has negligible penalty when $|\beta_2|B^2L < 0.05$

Lasers have $C = -4$ to -8 giving $\delta_c \approx 4$–6 dB when $|\beta_2|B^2L = 0.05$

A negative penalty occurs if $\beta_2 C < 0$ due to initial pulse compression



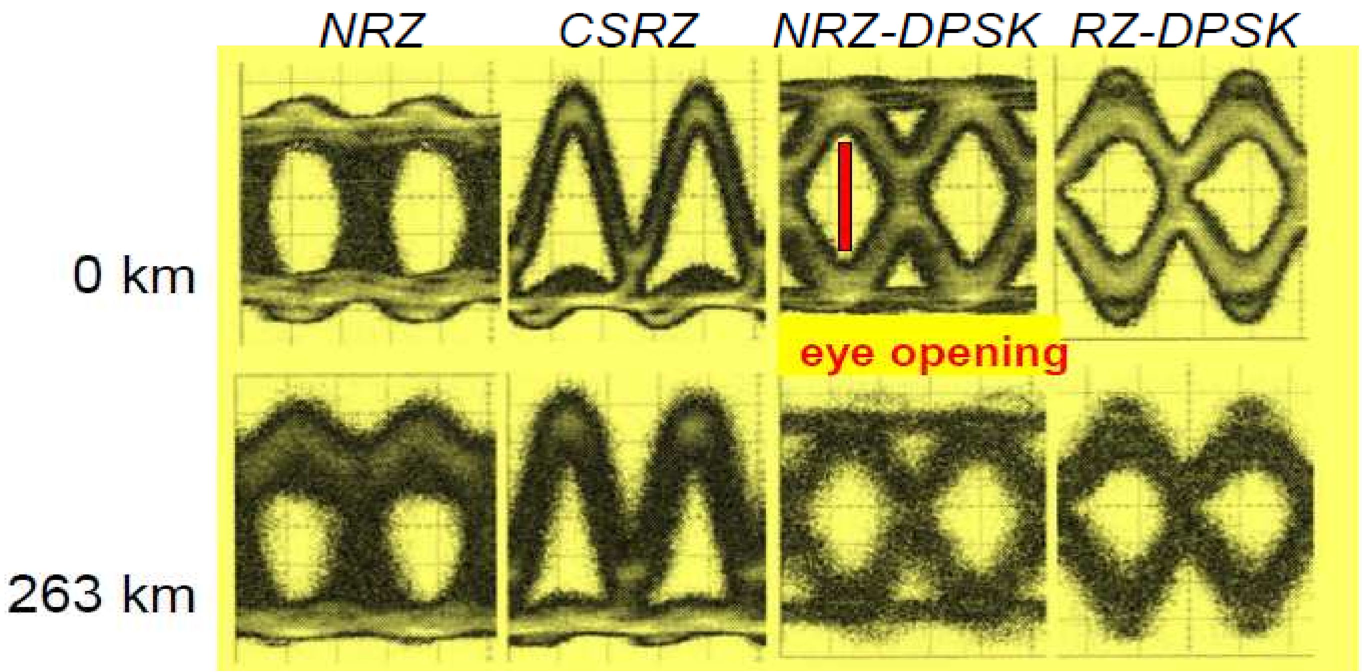
Eye-closure penalty

The eye is often used to monitor the signal quality

The ***eye-closure penalty*** is

$$\delta_{eye} = -10\log_{10}\left(\frac{\text{eye opening after transmission}}{\text{eye opening before transmission}}\right)$$

– This definition is ambiguous since "eye opening" is not well defined



Forward error correction (FEC)

FEC can correct errors and reduce the BER

Redundant data is introduced

- Decreases the ***effective bit rate...***
  - With given throughput, the bit rate increases
- ...but BER is typically decreased by this operation

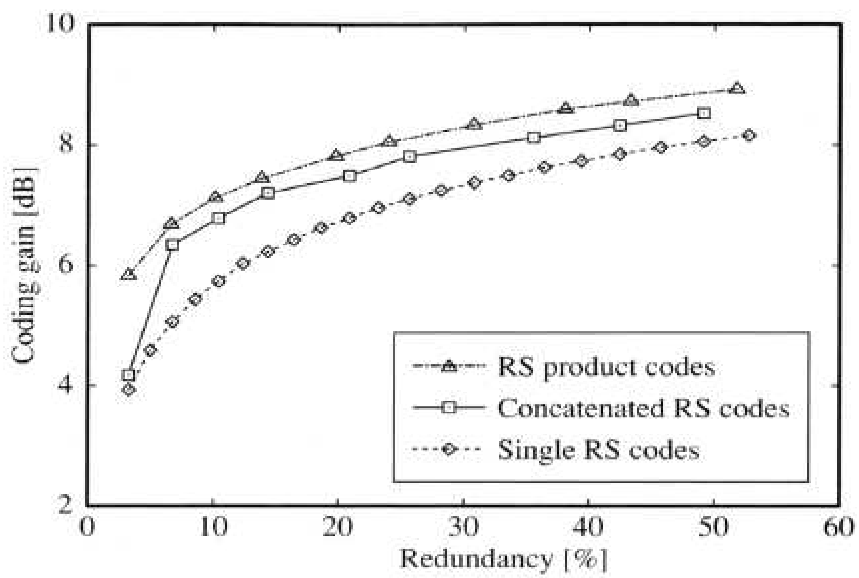
Increases system complexity since encoders/decoders are needed

Optical systems use simple FEC

- Symbol rate is very high,
  real-time processing is very difficult
- Reed-Solomon, RS(255, 239) is often used (gives 7% overhead)

Coding gain is here $G_c = 20\log_{10}(Q_c / Q)$

- Q_c is Q value when using FEC
- Coding gain of 5–6 dB is obtained with modest redundancy



Optimum FEC

The coding gain saturates with increasing redundancy

- There is an optimal redundancy depending on system parameters

Figure shows simulated *Q* values before and after FEC decoding

- WDM system, 25 channels, 40 Gbit/s per channel
- FEC increases system reach considerably