Measurements in sinusoidal steady state regime. Phase shift measurement.
rev. 1

Purpose: Familiarization with the methods of measurement of the features of the transfer function, and the representation of the frequency characteristics of a linear time-invariant (**LTI**) circuit. Using these measurements to determine the oscilloscope's input capacity and to study a compensated attenuator.

Summary of theory

By applying, to the input of a LTI circuit, a sinusoidal signal

$$x(t) = U_i \cdot \cos(\omega \cdot t) = \mathrm{Re}\{U_i \cdot e^{j\cdot\omega t}\}, \quad (1)$$

at the output of the circuit, a sinusoid, of the same frequency as the input signal, is obtained:

$$y(t) = U_0 \cdot \cos(\omega \cdot t + \varphi) = \mathrm{Re}\{H(\omega) \cdot U_i \cdot e^{j\cdot\omega t}\}, \quad (2)$$

where $H(\omega)$ is the value of the transfer function of the circuit at frequency f and $\omega = 2\pi f$.

As $H(\omega)$ is a complex number with the magnitude $|H(\omega)|$, and the argument $\arg\{H(\omega)\}$:

$$H(\omega) = |H(\omega)| \cdot e^{j\cdot\arg\{H(\omega)\}}, \quad (3)$$

the **amplitude** of the output signal, U_0, and the **phase shift** between the input signal and the output signal, φ, are obtained as follows:

$$U_0 = U_i \cdot |H(\omega)|, \qquad \varphi = \arg\{H(\omega)\} = arctg\left(\frac{\mathrm{Im}\{H(\omega)\}}{\mathrm{Re}\{H(\omega)\}}\right) \quad (4)$$

Formula (4) shows a way to determine both the magnitude and argument, which are also called the features of the transfer function.

The Magnitude Characteristics $|H(\omega)|$

The magnitude of the transfer function is measured at the frequency f_1, by applying to the input of a LTI circuit a sinusoid of frequency f_1 and known amplitude U_i. After measuring the amplitude of the output sinusoid, U_0, the magnitude of the transfer function, at that frequency, is determined with formula (5):

$$|H(\omega)| = \frac{U_0}{U_i} \quad (5)$$

If $|H(\omega)| > 1$, the circuit is said to *amplify*. If $|H(\omega)| < 1$, the circuit is said to *attenuate*.

$$amplification = |H(\omega)|, \quad attenuation = 1/|H(\omega)|, \quad (6)$$

It is more useful to express the magnitude of the transfer function in dB:

$$|H(\omega)|_{dB} = 20 \cdot \lg|H(\omega)| = 20 \cdot \lg\left(\frac{U_0}{U_i}\right) \quad (7.a)$$

$$|H(\omega)|_{dB} = 20 \cdot \lg\left(\frac{U_0}{U_{REF.}}\right) - 20 \cdot \lg\left(\frac{U_i}{U_{REF.}}\right) = U_0|_{dB} - U_i|_{dB} \quad (7.b)$$

The magnitude characteristic represents the variation of the magnitude of the transfer function with the frequency / pulsation ($\omega = 2\pi f$).

The magnitude characteristic can be graphically represented using a system of linear coordinates, semi-logarithmic coordinates or double logarithmic coordinates (Fig. 1). The third type of system is preferred. Double logarithmic system, called Bode diagram, allows the representation of the amplitude characteristics in a wide range of pulsations, i.e. frequencies.

The angular frequency (pulsation) range, between an arbitrary value ω_1, and $10\omega_1$, is called ***decade***, and the range between ω_1 and $2\omega_1$, is called ***octave.***

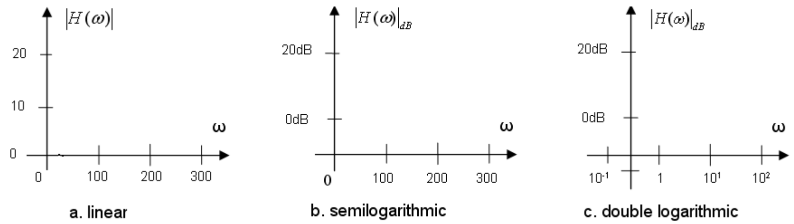

Fig. 1

For the magnitude characteristic, of particular importance is the cutoff frequency, f_{-3dB}, which is defined as the frequency at which the power of the output sinusoid is equal to half the maximum power possible (in frequency), provided that the applied input signal is sinusoidal. At this frequency the magnitude of the transfer function is 3 dB smaller than its maximum value (expressed in dB).

$$|H(\omega_{-3dB})|_{dB} = \max_{\omega}\{|H(\omega)|_{dB}\} - 3$$
$$|H(\omega_{-3dB})| = \frac{\max_{\omega}\{|H(\omega)|\}}{\sqrt{2}} \cong 0.707 \cdot \max_{\omega}\{|H(\omega)|\} \quad (8)$$

A 3 dB reduction of the magnitude of the transfer function, expressed in dB, is equivalent to a reduction by $\sqrt{2}$ of the value of the modulus.

The Phase Characteristics $\arg\{|H(\omega)|\}$

By applying a sinusoid, of frequency f_1 and known amplitude U_i, at the input of a LTI circuit, and measuring the phase shift between the output and the input signals, the argument of the transfer function, at that frequency, $\arg\{H(\omega)\}$, is obtained.

The plot of the variation of the phase shift with the frequency/pulsation, introduced by the circuit, is called *the phase plot*.

The phase plot can be measured with the scope through two simple methods: the *ellipse method* and the *synchronization with the reference signal method*. It also can be measured with devices called phase-meters.

The Ellipse Method

The ellipse method is applicable for a two-channel scope. This should be done at an assembly as in Fig. 2.a. Changing the working mode of the scope in $Y(X)$, the image obtained, on the display of the scope, is an ellipse with axes rotated relative to the coordinate system, as shown in Figure 2.b.

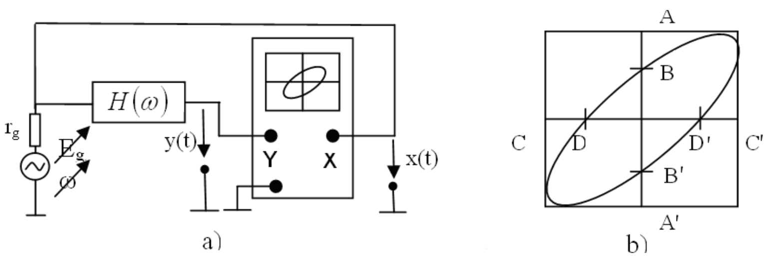

Fig. 2. a) measuring assembly, b) the image on the scope.

The parametric equations of the ellipse are:

$$\begin{cases} dx = K_x U_i \cos(\omega t) \\ dy = K_y \left| H(\omega) \right| \cdot U_i \cdot \cos(\omega t + \varphi + \varphi_{xy}) \end{cases} \quad (9)$$

where

- dx, dy represent the spot deviation on the display of the scope, in direction OX, respectively OY;
- K_x, K_y are the deviation coefficients (of the spot) corresponding to input X, respectively Y;
- $\varphi = \arg\{H(\omega)\}$ represents the phase shift between the two signals;
- φ_{xy} is phase shift between the two channels (X, Y) of the scope (oscilloscope internal phase shift).

The scope internal phase shift can be measured by applying the same signal on both inputs, CH1 and CH2. If the image which appears in the $Y(X)$ mode, is a line, given by the equation (10), the phase shift between the channels can be neglected.

$$dy = \frac{K_y}{K_x} \cdot dx \quad (10)$$

The line segments read from the display (Figure 2b), in order to determine the phase shift angle, are presented in terms of their meaning, manner to measure them, and expressions obtained from parametric equations.

- **AA'** – the distance between tangents parallel to OX (peak to peak amplitude of dy). The length of the segment appearing on the display when disconnecting the signal from the input X, is measured.

$$AA' = 2 \cdot K_y \cdot \left| H(\omega) \right| \cdot U_i \quad (11)$$

- **CC'** – the distance between the tangents to the ellipse parallel to OY (peak to peak amplitude of dx). The length of the segment appearing on the display when disconnecting the signal from the input Y, is measured.

$$CC' = 2 \cdot K_x \cdot U_i \quad (12)$$

- **BB'** – the distance between the points of intersection of the ellipse with OY (twice the instantaneous value of dy, when $dx = 0$). It is measured on the ellipse.

$$BB' = 2 \cdot K_y \cdot \left| H(\omega) \right| \cdot U_i \cdot \left| \sin\varphi \right| \quad (13)$$

- **DD'** – the distance between the points of intersection of the ellipse with OX (twice the instantaneous value of dx, when $dy = 0$). It is measured on the ellipse.

$$DD' = 2 \cdot K_x \cdot U_i \cdot \left| \sin\varphi \right| \quad (14)$$

Using the values of the segments, the following expression is obtained:

$$\left| \sin\varphi \right| = \frac{BB'}{AA'} = \frac{DD'}{CC'} = \lambda \quad (15)$$

Since in the relations for CC' and DD', $\left| H(\omega) \right|$ (which changes with the frequency) does not appear, for determining λ, the ratio bellow is preferred:

$$\lambda = \frac{DD'}{CC'} \quad (16)$$

From the formula (14), taking into account the position of the major axis of the ellipse, the phase shift between the input signal and the output one of LTI circuit, having the transfer

function $H(\omega)$, can be determined. This phase shift corresponds to the argument of the transfer function of the circuit at the frequency ω (if $\varphi_{xy} = 0$).

If the major axis of the ellipse is in the first quadrant, then the phase shift is calculated using the expression:

$$\varphi = \pm \arcsin\lambda \quad (17)$$

If major axis of the ellipse is in the second quadrant, then the phase shift is calculated using the expression:

$$\varphi = \pi \pm \arcsin\lambda \quad (18)$$

The sign is resolved by introducing an additional phase shift, of known value, on one of the channels and observing the way the ellipse changes.

Note: The ellipse method is not recommended when $\varphi \in \{k\pi + \pi/2\}, k \in Z$.

The 'Synchronization with Reference Signal' Method

This method can be used both with a scope with two channels, and a scope with a single channel.
The measuring assembly is presented in Figure 3a; here X and Y are the names of the 2 chanels, showing signals *x(t)* and *y(t)*, but the scope is no longer in X-Y display mode. In Figure 3b the image that appears on the oscilloscope screen, set in the Y(*t*) mode is shown.

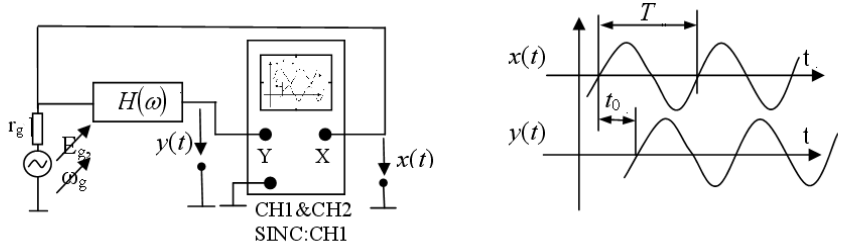

Fig. 3. Phase shift measuring using the synchronization method for an oscilloscope with two channels.

The signal $y(t)$, the output of the circuit, can be written as:

$$y(t) = U_0 \cdot \cos(\omega \cdot t + \varphi) = U_0 \cdot \cos(\omega \cdot (t + t_0)) \quad (19)$$

where

$$\varphi = \omega \cdot t_0 = 2 \cdot \pi \cdot \frac{t_0}{T} = 360^\circ \cdot \frac{t_0}{T} \quad (20)$$

T represents the signal's period, and t_0 is the difference in time between zero crossings, with the same slope, of signals $y(t)$ and $x(t)$. Measuring values t_0 and T, and using the expression (20), the phase shift can be determined.

Remark: To measure time ranges with minimum error, using the scope, their demarcation moments are chosen to be the moments when the slope of the signals ($x(t)$ ***or*** $y(t)$ *)* reaches its maximum.

Remark:This method can be used for a single channel scope (CH1), using the possibility of **external triggering** of the scopes. Thus, after establishing the conditions for triggering using signal $x(t)$, it is applied on **External Trigger**, and $y(t)$ on CH1. In this case, t_0 is the difference between the *trigger moment* of the scope and the time at which the signal $y(t)$ satisfies the trigger conditions.

The Oscilloscope: the Lissajous figure, the input impedance

Remember the oscilloscope has two display modes:

- $Y(t)$ mode - the temporal variation of the signal, applied on one of the channels of the scope, is viewed. In this mode, the signal commands the movement of the spot on the Oy axis of the display. The moving of the spot on the Ox axis is given by the time base.
- XY mode – the spot moving on Ox axis of the screen is no longer commanded by the time base, but by the signal applied to the second input of the scope. For this working mode, it is necessary for the scope to have two inputs. If the two signals are periodic, with $T_y = M \cdot T_0$ and $T_x = N \cdot T_0$ (M, N *integers*), at intervals $T = M \cdot N \cdot T_0$, $u_x(k \cdot T)$ and $u_y(k \cdot T)$ have the same values (k integer). Therefore, the spot describes a closed curve called Lissajous figure (Figure 4).

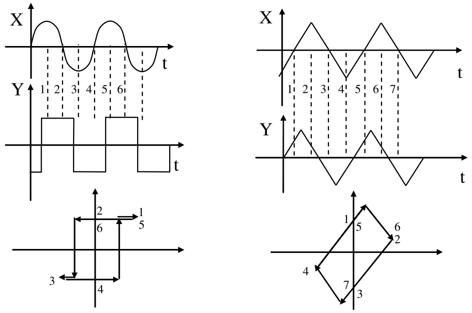

Fig. 4. Example of Lissajous figures

If sinusoidal signals of equal frequency are applied on the two channels, an ellipse, having geometric properties defined by the phase shift between the two sinusoids, is obtained on the display. Also, in case sinusoids have frequencies whose ratio is an integer, some particular images are obtained: (Figure 5).

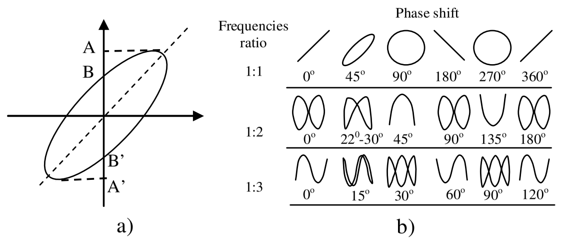

Fig. 5. Lissajous images – on X,Y phase shifted sinusoids are applied.

The oscilloscope's input impedance

It has the structure in Figure 6:

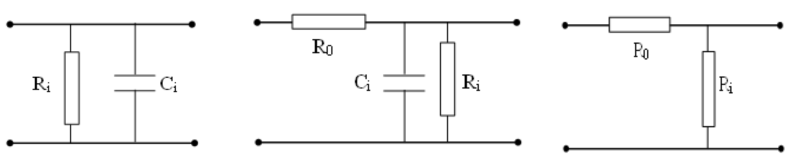

Fig. 6. Z_i impedance Fig. 7. Low pass filter Fig. 8. Resistive divider

To determine the values of the input resistor and capacitor of the scope, an additional resistor is inserted, in series on the scope's input. A low-pass filter (Fig. 7) is formed. It has a transfer characteristic given by:

$$H(\omega) = \frac{R_i}{R_0 + R_i} \cdot \frac{1}{1 + j \cdot \omega \cdot C_i \cdot R_0 \parallel R_i} \quad (21)$$

For low frequencies, the input capacity can be neglected; and the circuit becomes a simple resistive divider (Fig. 8), with the transfer function

$$H(\omega) = \frac{R_i}{R_0 + R_i} \quad (22)$$

By using the oscilloscope, the amplitudes of the input and the output signals are measured, and the two elements of the input impedance can be determined.

Calibrated attenuator:

The calibrated attenuator is used in order to obtain the attenuation steps, necessary to obtain calibrated deflection coefficients. Attenuators could be made as resistive dividers, but taking into account the restrictions imposed (there exists input capacity C_i, which can not be neglected), for the attenuator, an additonal capacitor C_a has to be inserted; a schematic equivalent to the one shown in Fig. 9 is obtained.

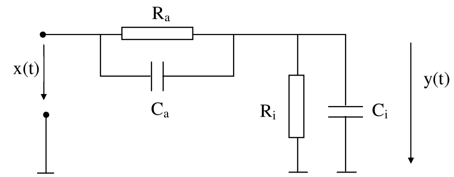

Fig. 9. The structure of a attenuator.

The transfer function of the circuit is given by:

$$H(\omega) = \frac{Z_i(\omega)}{Z_i(\omega) + Z_a(\omega)} \quad (23)$$

where

$$Z_i(\omega) = R_i \parallel \frac{1}{j\omega C_i} = \frac{R_i}{1 + j\omega R_i C_i}$$
$$Z_a(\omega) = R_a \parallel \frac{1}{j\omega C_a} = \frac{R_a}{1 + j\omega R_a C_a} \quad (24)$$

It can be observed that, if $R_i \cdot C_i = R_a \cdot C_a$, the transfer function becomes

$$H(\omega) = \frac{R_i}{R_i + R_a} = H_0 = k \quad (25)$$

i.e. independent of frequency. "$R_i \cdot C_i = R_a \cdot C_a$" is called the **condition for compensation**, and C_a is also called compensation capacitor. The signal step response of the attenuator will also be a signal step, $\sigma(t)$, multiplied by the value k, $y(t) = k \cdot \sigma(t)$. The rectangular signal will be reproduced accurately, regardless of frequency.

If $R_i \cdot C_i \neq R_a \cdot C_a$, by substituting, in (1), the expressions of the impedances $Z_i(\omega)$ and $Z_a(\omega)$, from relations (2), for $H(\omega)$ one obtains the expression:

$$H(\omega)=\frac{R_i\cdot(1+j\cdot\omega\cdot R_a\cdot C_a)}{R_i+R_a+j\cdot\omega\cdot R_i\cdot R_a\cdot(C_i+C_a)} \qquad (26)$$

In this case, for the output signal of the attenuator, one obtains:

$$y(t)=\frac{R_i}{R_i+R_a}\cdot\sigma(t)+\frac{\tau_i-\tau_a}{(C_i+C_a)\cdot(R_i+R_a)}\cdot\left(1-e^{-\frac{1}{\tau_i}t}\right)\cdot\sigma(t) \qquad (27)$$

where

$$\tau_a=R_a\cdot C_a$$
$$\tau_i=\frac{R_i\cdot R_a\cdot(C_i+C_a)}{R_i+R_a} \qquad (28)$$

Depending on the relationship between τ_i and τ_a, the second term is positive or negative:

$\tau_i>\tau_a$ undercompensated attenuator

$\tau_i<\tau_a$ overcompensated attenuator

In Fig.10, the response of the attenuator is presented, in three cases (compensated attenuator, over-compensated attenuator, and under-compensated attenuator).

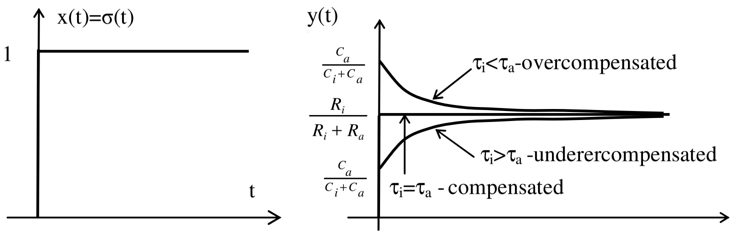

Fig. 10. The response of the attenuator.

Measurements

NOTE: Students will bring calculators with trigonometric functions!

Remark 1. The scope should be reset to its default state by clicking **Default/Setup**. The same is done for the function generator, **Shift+2**=*Default.*

Remark 2. Since we are not using any divider probe on the oscilloscope, for all measurements the setting **Probe=1x** (CH1 Menu and CH2 Menu) must be used. Otherwise, the values measured and indicated by the oscilloscope would be wrong (*probe* times bigger).

1. *Measuring the cutoff frequency (f_{-3dB}) of the filter*

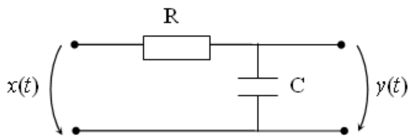

Fig. 11

a) measure the R and C components, using the digital multimeter. Make the circuit corresponding to Fig. 11, on the test breadboard. Input to the circuit (from the function

generator) a sinusoid of frequency $f_1=100Hz$, without DC bias (verify that the rotary OFFSET knob is pressed, not pulled), with the level (amplitude) set to $U_i|_{dB}=0dB$, measured on the dB scale of the AC millivoltmeter. Also measure the level of the output signal of the circuit, $U_o|_{dB}$, on the dB scale of the AC millivoltmeter. Modify the frequency of the signal until it reaches the cutoff frequency, f_{-3dB} (frequency at which the output voltage is 3dB lower than the input voltage. Since $U_i|_{dB}=0dB$, the frequency f_{-3dB} is obtained when $U_o|_{dB}=-3dB$).

Calculate the theoretical value with the formula $f_{-3dB}=1/(2\cdot\pi\cdot R\cdot C)$.

b) Determine the ratio between the amplitudes of the output and the input of the circuit, at the frequency f_{-3dB}, with the oscilloscope. To determine the amplitude, you can use the cursors of the scope. How much should this value be (from theory)?

2. *Measurement of the magnitude-frequency transfer function.*

a) Determine the modulus of the transfer function of the $R-C$ filter (Fig. 11). Input to the circuit a sinusoid of frequency f_i, having the level (amplitude) set at $U_i|_{dB}=0dB$. Measure the level of the output signal, $U_o[dB]$, on the dB scale of the AC millivoltmeter. Calculate the modulus of the transfer function (relation $(7.b)$), $|H(\omega)|_{dB}=U_o|_{dB}-U_i|_{dB}$. Measure at fequencies $f_{-3dB}/10$, $f_{-3dB}/4$, $f_{-3dB}/\sqrt{3}$, f_{-3dB}, $\sqrt{3}\cdot f_{-3dB}$, $4\cdot f_{-3dB}$, $8\cdot f_{-3dB}$, $10\cdot f_{-3dB}$, $40\cdot f_{-3dB}$, where f_{-3dB} is the frequency measured at 1a).

b) From measurements made at 2.a), determine the slope of the filter in the stop band (the range of frequencies higher than f_{-3dB}). Calculate the slope of the filter in dB/decade, and in dB/octave (with how many decibels is the amplitude decreasing when the frequency of the signal is increased 10 times, 2 times, respectively).

3. *Measurement of phase transfer function*

Measure the phase shift, using the scope, through the ellipse method and through the synchronization with the reference signal method, at the frequencies: $f_{-3dB}/10$, $f_{-3dB}/4$, $f_{-3dB}/\sqrt{3}$, f_{-3dB}, $\sqrt{3}\cdot f_{-3dB}$, $4\cdot f_{-3dB}$, $10\cdot f_{-3dB}$. Write down the values (*in degrees*) in Table 2 on your worksheets. Use the value you measured at 1a), for the cutoff frequency.

			The ellipse method				The ref. signal synchronization method						
f	f $[kHz]$	φ_{th} $[^\circ]$	DD'	CC'	φ_e	ε_1	t_0	T	φ_{synch}	ε_2	ε_{t_0}	ε_T	ε_φ
$f_{-3dB}/10$													
$f_{-3dB}/4$													
$f_{-3dB}/\sqrt{3}$													
f_{-3dB}													
$\sqrt{3}f_{-3dB}$													
$4f_{-3dB}$													
$10f_{-3dB}$													

Table 2

φ_t – the phase shift (in degrees), determined according to:

$$\varphi_{th.} = -arctg\left(\frac{f}{f_{-3dB}}\right) \tag{29}$$

φ_e – the phase shift measured through the ellipse method.

φ_{synch} – the phase shift measured through the synchroniztion method using the two channel scope.

Measurement using the ellipse method

a) Input the sinusoid $x(t)$, from the function generator, at CH1, and the signal $y(t)$, from the output of the circuit (Fig. 11), at CH2 of the scope. Select the display mode for the scope **Display→Y(X),** and, in the absence of the two signals (coupling **GND** for CH1 and CH2), centrally position the point wich appears on the display. Apply the signals on both channels (coupling **DC** for CH1 and CH2), and adjust the vertical deflection coefficients at the same value ($C_{y_1} = C_{y_2}$), chosing their value so that the image of the ellipse is as large as possible on the display. Measure the segments CC' and DD' on the display. Write down their values in the table, and calculate the phase shift ($\varphi_e = -\arcsin\lambda$, where $\lambda = \frac{DD'}{CC'}$). Do these measurements at each frequency mentioned in the table.

Remark: In order to simplify the measurements, note that the absolute values of the segments DD' and CC' are not important, only the ratio between them is taken into acount. Therefore, before each measurement, you can adjust the amplitude from the generator until the ellipse "fills" the whole display, ie each time CC' is as big as possible. You can read CC' and DD' in divisions. You can easily measure CC' by temporarily disconnecting the signal from CH2 of the scope.

Measurement using the synchronization method

b) Measure the phase shift, using the synchronization with reference signal method, with the two-channels scope. Input sinusoidal signal $x(t)$ to CH1, and the signal $y(t)$ at CH2 of the scope. Set the oscilloscope in the display mode **Display→ Y(t),** and position the zero level in the middle of the screen, for both channels (coupling **GND** for CH1 and CH2). Then, apply the signals on the two channels (coupling DC for CH1 and CH2), trigger on CH1, AC coupling (the **Trigger** menu). For a more precise measurement, adjust the amplitude so that the image is as large as possible on the display. Using time cursors, measure t_0 and T, according to figure $(3.b)$. Also measure $\varepsilon_{t_0} = \frac{\Delta t}{t_0}$ and $\varepsilon_T = \frac{\Delta t}{T}$ (Δt is the smallest change of the indication of the time cursor, on the current image). Calculate the phase shift, $\varphi_{synch} = -360^\circ \cdot \frac{t_0}{T}$ and write it downn in Table 2 for all the frequencies required.

c) Calculate the relative error, done when determining the phase shift, comparing to the theoretical value.

$$\varepsilon_1 = \frac{\varphi_{th} - \varphi_e}{\varphi_{th}}; \quad \varepsilon_2 = \frac{\varphi_{th} - \varphi_{synch}}{\varphi_{th}}; \tag{30}$$

d) Calculate the maximum error due to the reading on the screen of the scope, for the reference signal synchronization method, using the relative errors done when reading, calculated at b).

Calculate the error, for the synchronization method, with the relation (32):

$$\varphi = 360^\circ \cdot \frac{t_0}{T} \tag{31}$$

$$\varepsilon_\varphi = \varepsilon_{t_0} + \varepsilon_T \tag{32}$$

Remark: Calculate the error, for the ellipse method, with relation (33):

$$\varepsilon_\varphi = \frac{1}{\sqrt{1-\left(\frac{DD'}{CC'}\right)^2}} \cdot \frac{DD'}{CC'} \cdot (\varepsilon_{DD'} + \varepsilon_{CC'}) \tag{33}$$

4. The *Bode diagram for the magnitude and the phase characteristics*

Graphically represent the magnitude and the phase characteristics, for the studied circuit, using the Bode plots (with double-logarithmic scales, as in Fig. 1.c.). Use the values obtained with the ellipse method, for the phase characteristics.

5. Obtaining Lissajous figures with the scope

Set the alternative function, TTL output, from the generator, using the SHIFT key followed by the TTL softkey, so that the **TTL** indicator is lit on the display. This output generates a signal at the same frequency as the main output, but rectangular and with TTL level. Apply the signal, from the TTL output of the generator, to the input of the circuit in Fig. 11. The frequency of the signal will be $10 \cdot f_{-3dB}$, where f_{-3dB} is the cutoff frequency of the circuit, determined at **1**. Apply the signal from the output of the circuit, to the CH1 input of the scope. Draw the image obtained on the display. What is the function of the circuit?

On CH2, input a triangular signal, obtained from the output of the generator. Draw this image on the same graph.

Set the working mode of the scope to $X(Y)$, and draw the obtained image, called Lissajous figure. ***Note:*** *the ellipsis obtained in Section 3 is a special case of Lissajous figure.*

Change the waveform from sinusoidal into triangular. Draw the new image.

Applications of the measurement in sinusoidal steady state regime

6. Measurement of the oscilloscope's input capacitor

Measure the input capacitor C_i for CH1. Use a sinusoidal test signal. Apply this signal to CH1, through a high value resistance R_o, inserting it between the two "alligators" located on the red cables (previously, measure its value, on the ohmmeter). Measure once at a low frequency ($f = 500Hz$), and adjust the amplitude U_{20} (for ease of measurement , set it to be 4 divisions, to "fill" the full screen). Increase the frequency of the generator until the voltage indicated on the scope drops by 3dB compared to U_{20}. Write down this frequency, f_s', which represents the cutoff frequency of the circuit formed by the resistance R_o *th*e input impedance oscilloscope, R_i in parallel with C_i. Consider the value of the input resistance to be of 1MΩ.

When calculating C_i, take into account that

$$f_s' = \frac{1}{2 \cdot \pi \cdot (R \parallel R_i) \cdot C_i} \tag{34}$$

7. Measurement of a attenuator

Assemble the attenuator in Fig. 12, on the test board:

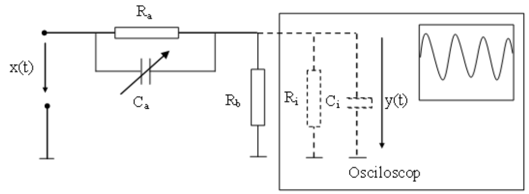

Fig. 12. Measuring schematic for the attenuator.

Choose the following values : $R_a = 178k\Omega,\ R_b = 39k\Omega,$ and C_a is a variabile capacitor (trimmer).

Input a rectangular signal, of frequency $100kHz$ and the amplitude of $5V$, from the function generator. Adjust the amplitude of the signal, connecting the generator directly to the scope. Then, input the signal, from the generator, to the attenuator.

a) Adjust the variable capacitor with the screwdriver so that the attenuator is compensated (on the display, you must obtain undistorted rectangular signal - see Fig. 10). Calculate the attenuation of the signal on the oscilloscope, calculating the ratio between the amplitude of the input signal, and the amplitude measured on the display of the scope, after passing through the attenuator. Calculate the attenuation, from expression (25). Since R_b has a very small value, comparing to the oscilloscope's input resistor R_i, the value of the resistor of the scope can be neglected, when calculating the attenuation.

Draw the image you obtain on the display, for the compensated attenuator.

b) Using the compensation condition, deduce the value of the variable capacitor. For C_i consider the value determined at **6**.

c) Adjust the variable capacitor C_a, so that on the display of the scope, the overshoot caused by the overcompensation, is at least of one division. Measure the value of the overshoot. Calculate the value of the overshoot, with the relation specified on the graphical representation of the response of the attenuator at rectangular signal (see Fig. 10). Compare it with the measured value. Draw the image, in the case of overshoot.

d) Adjust the variable capacitor so that the attenuator is undercompensated. Draw the obtained image.

Preparatory questions

1. Briefly describe the ellipse method, specifying the formula for calculating the phase angle.
2. Briefly describe the synchronization with reference signal method, specifying the formula for calculating the phase angle.
3. Define the frequency f_{-3dB}.
4. Determine the value $U_1 / U_2 \mid dB$, if $U_1 / U_2 = 20.$
5. Determine the value U_1 / U_2, if $U_1 / U_2 \mid dB = 34dB.$

Tip: $\lg 2 \approx 0.3, \lg 3 \approx 0.477, \lg 5 \approx 0.7.$

6. For which values of the phase angle, the ellipse method is not recommended ? Why?
7. What is a Bode diagram? Draw the diagram corresponding to a low pass filter.
8. What does a decade of frequencies represent ? What about an octave?
9. How to choose the moments of time in order to measure the period of the signal $x(t) = U_i \cdot \cos(\omega t)$, using the scope ? Justify.
10. Determine the voltage in V, if $U\,|_{dB} = +20dBm.$
11. Determine the value of the voltage in dB, if $U = 0.1V$.

12. For the circuit with the transfer function $H(\omega) = \dfrac{1}{1 + j\omega RC}$, determine $|H(\omega)|,\ \arg\{H(\omega)\},$ $\max\limits_{\omega}\{|H(\omega)|\}$, and the formula for f_{-3dB}.
13. Deduce the transfer function (magnitude and phase) of the circuit in Fig. 11.
14. Determine the frequency f_{-3dB} for an oscilloscope with $R_i = 1M\Omega$ and $C_i = 30pF$, if a resistor R is used, to attenuate 10 times.
15. Demonstrate the formula for the phase error in the synchronization method ($\varepsilon_\varphi = \varepsilon_{t_0} + \varepsilon_T$, with $\varphi = 360^\circ \cdot \dfrac{t_0}{T}$).
16. The phase shift introduced by a linear circuit is measured using the synchronization method. The delay between the output and the input is $t_0 = 250\mu s$, and the frequency of the signals is $1kHz$. Determine the phase shift introduced by the circuit.
17. A compensated attenuator has $R_a = 1M\Omega,\ C_a = 80pF$, and $R_i = 2M\Omega$. Determine the value of the capacitor C_i.
18. A sinusoidal signal is input to a scope, through a resistor $R_0 = 2M\Omega$. The frequency of the signal is $f_0 = 100Hz$. If the input signal has the amplitude $A = 4V$, and the signal amplitude measured on the display of the scope is of $1V$, determine the input resistance of the scope (the effect of the input capacity is neglected).
19. Deduce the transfer function (magnitude and phase) of the circuit in Fig. 9.

Homeworks

Homework 1: Determine the magnitude and the argument of the transfer function for the circuits in Fig. 13, at the frequency $f = 20/\pi\ kHz.$

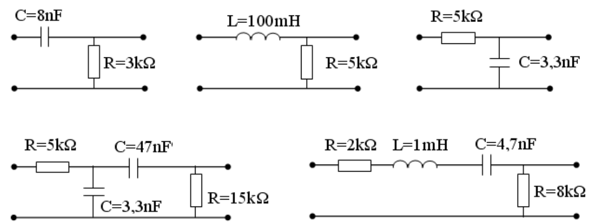

Fig. 13

Determine the cutoff frequency for these circuits. Plot asymptotic magnitude and phase characteristics.

Homework 2: For the circuit in Fig. 14, determine and graphically represent the magnitude and the argument of the transfer function ($Z(\omega) = U(\omega) / I(\omega)$). Determine the frequency of resonance.

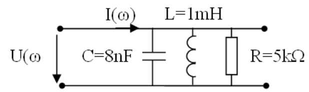

Fig. 14

Homework 3: Deduce the relation (33). Tip: Start from the error formula for indirect measurements; if $y = f(x_1, x_2, ..., x_n)$, then:

$$\varepsilon^y = \frac{1}{y} \cdot \sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \cdot x_i \cdot \varepsilon^{x_i} \right| \qquad (33)$$